Simplification and Factorisation

1.3

Introduction

In this Section we explain what is meant by the phrase 'like terms' and show how like terms are collected together and simplified.

Next we consider removing brackets. In order to simplify an expression which contains brackets it is often necessary to rewrite the expression in an equivalent form but without any brackets. This process of removing brackets must be carried out according to particular rules which are described in this Section.
Then factorisation, which can be considered as the reverse of the process, is dealt with. It is essential that you have had plenty practice in removing brackets before you study factorisation.

Prerequisites

Before starting this Section you should ...

① be familiar with algebraic notation

✓ be able to use the laws of indices

Learning Outcomes

After completing this Section you should be able to ...

✓ collect together like terms in order to be able to simplify expressions

✓ be able to simplify expressions by collecting like terms

✓ be able to use the laws of indices

✓ identify common factors in an expression

✓ factorise simple expressions

✓ factorise quadratic expressions

1. Addition and subtraction of like terms.

Like terms are multiples of the same quantity. For example $5y$, $17y$ and $\frac{1}{2}y$ are all multiples of y and so are *like terms*. Similarly, $3x^2$, $-5x^2$ and $\frac{1}{4}x^2$ are all multiples of x^2 and so are like terms.

Further examples of like terms are:

kx and ℓx which are both multiples of x,

x^2y, $6x^2y$, $-13x^2y$, $-2yx^2$, which are all multiples of x^2y

abc^2, $-7abc^2$, $kabc^2$, are all multiples of abc^2

Like terms can be added or subtracted in order to simplify them.

Example Simplify $5x - 13x + 22x$.

Solution

All three terms are multiples of x and so are like terms. The expression can be simplified to $14x$.

Example Simplify $5z + 2x$.

Solution

$5z$ and $2x$ are not like terms. They are not multiples of the same quantity. This expression cannot be simplified.

Simplify $5a + 2b - 7a - 9b$

Your solution

$5a + 2b - 7a - 9b =$

$-2a - 7b$

Example Simplify $2x^2 - 7x + 11x^2 + x$.

Solution

$2x^2$ and $11x^2$, both being multiples of x^2 can be collected together and added to give $13x^2$. Similarly, $-7x$ and x are like terms and these can be added to give $-6x$. We find $2x^2 - 7x + 11x^2 + x = 13x^2 - 6x$ which cannot be simplified further.

Simplify $\frac{1}{2}x + \frac{3}{4}x - 2y$

Your solution

$\frac{1}{2}x + \frac{3}{4}x - 2y =$

$\frac{5}{4}x - 2y$

Example Simplify $3a^2b - 7a^2b - 2b^2 + a^2$.

Solution

Note that $3a^2b$ and $7a^2b$ are both multiples of a^2b and so are like terms. There are no other like terms. Therefore

$$3a^2b - 7a^2b - 2b^2 + a^2 = -4a^2b - 2b^2 + a^2$$

Exercises

1. Simplify, if possible,

   a) $5x + 2x + 3x$, b) $3q - 2q + 11q$, c) $7x^2 + 11x^2$, d) $-11v^2 + 2v^2$, e) $5p + 3q$

2. Simplify, if possible, a) $5w + 3r - 2w + r$, b) $5w^2 + w + 1$, c) $6w^2 + w^2 - 3w^2$

3. Simplify, if possible,

   a) $7x + 2 + 3x + 8x - 11$, b) $2x^2 - 3x + 6x - 2$, c) $-5x^2 - 3x^2 + 11x + 11$,

   d) $4q^2 - 4r^2 + 11r + 6q$, e) $a^2 + ba + ab + b^2$, f) $3x^2 + 4x + 6x + 8$,

   g) $s^3 + 3s^2 + 2s^2 + 6s + 4s + 12$.

4. Explain the distinction, if any, between each of the following expressions, and simplify if possible.

   a) $18x - 9x$, b) $18x(9x)$, c) $18x(-9x)$, d) $-18x - 9x$, e) $-18x(9x)$

5. Explain the distinction, if any, between each of the following expressions, and simplify if possible.

   a) $4x - 2x$, b) $4x(-2x)$, c) $4x(2x)$, d) $-4x(2x)$, e) $-4x - 2x$, f) $(4x)(2x)$

6. Simplify, if possible,

   a) $\frac{2}{3}x^2 + \frac{x^2}{3}$, b) $0.5x^2 + \frac{3}{4}x^2 - \frac{11}{2}x$, c) $3x^3 - 11x + 3yx + 11$,

   d) $-4\alpha x^2 + \beta x^2$ where α and β are constants.

Answers

1. a) $10x$, b) $12q$, c) $18x^2$, d) $-9v^2$, e) cannot be simplified.
2. a) $3w + 4r$, b) cannot be simplified, c) $4w^2$
3. a) $18x - 9$, b) $2x^2 + 3x - 2$, c) $-8x^2 + 11x + 11$, d) cannot be simplified,
   e) $a^2 + 2ab + b^2$, f) $3x^2 + 10x + 8$, g) $s^3 + 5s^2 + 10s + 12$
4. a) $9x$, b) $162x^2$, c) $-162x^2$, d) $-27x$, e) $-162x^2$
5. a) $4x - 2x = 2x$, b) $4x(-2x) = -8x^2$, c) $4x(2x) = 8x^2$, d) $-4x(2x) = -8x^2$,
   e) $-4x - 2x = -6x$, f) $(4x)(2x) = 8x^2$
6. a) x^2, b) $1.25x^2 - \frac{11}{2}x$, c) cannot be simplified, d) $(\beta - 4\alpha)x^2$

2. Removing brackets from the expressions $a(b+c)$ and $a(b-c)$

Removing brackets means *multiplying out*. For example $5(2+4) = 5\times 6 = 30$ but also $5(2+4) = 5\times 2 + 5\times 4 = 10 + 20 = 30$. That is:

$$5(2+4) = 5\times 2 + 5\times 4$$

In an expression such as $5(x+y)$ it is intended that the 5 multiplies both x and y to produce $5x+5y$. Thus the expressions $5(x+y)$ and $5x+5y$ are equivalent. In general we have the following rules known as **distributive laws**:

Key Point

$$a(b+c) = ab + ac, \qquad a(b-c) = ab - ac$$

Note that when the brackets are removed both terms in the brackets are multiplied by a

As we have noted above, if you insert numbers instead of letters into these expressions you will see that both left and right hand sides are equivalent. For example

$$4(3+5) \text{ has the same value as } 4(3)+4(5), \text{ that is } 32$$

and

$$7(8-3) \text{ has the same value as } 7(8)-7(3), \text{ that is } 35$$

Example Remove the brackets from a) $9(2+y)$, b) $9(2y)$.

Solution

a) In the expression $9(2+y)$ the 9 must multiply both terms in the brackets:

$$\begin{aligned} 9(2+y) &= 9(2) + 9(y) \\ &= 18 + 9y \end{aligned}$$

b) Recall that $9(2y)$ means $9\times(2\times y)$ and that when multiplying numbers together the presence of brackets is irrelevant. Thus $9(2y) = 9\times 2\times y = 18y$

The crucial distinction between the role of the factor 9 in the two expressions $9(2+y)$ and $9(2y)$ should be noted.

Example Remove the brackets from $9(x + 2y)$.

Solution

In the expression $9(x + 2y)$ the 9 must multiply both the x and the $2y$ in the brackets. Thus

$$\begin{aligned} 9(x + 2y) &= 9x + 9(2y) \\ &= 9x + 18y \end{aligned}$$

Example Remove the brackets from $-3(5x - z)$.

Solution

The number -3 must multiply both the $5x$ and the z.

$$\begin{aligned} -3(5x - z) &= (-3)(5x) - (-3)(z) \\ &= -15x + 3z \end{aligned}$$

Example Remove the brackets from $6x(3x - 2y)$.

Solution

$$\begin{aligned} 6x(3x - 2y) &= 6x(3x) - 6x(2y) \\ &= 18x^2 - 12xy \end{aligned}$$

Example Remove the brackets from $-(3x + 1)$.

Solution

Although the 1 is unwritten, the minus sign outside the brackets stands for -1. We must therefore consider the expression $-1(3x + 1)$.

$$\begin{aligned} -1(3x + 1) &= (-1)(3x) + (-1)(1) \\ &= -3x + (-1) \\ &= -3x - 1 \end{aligned}$$

Example Remove the brackets from $-(5x - 3y)$.

Solution

$-(5x - 3y)$ means $-1(5x - 3y)$.

$$\begin{aligned} -1(5x - 3y) &= (-1)(5x) - (-1)(3y) \\ &= -5x + 3y \end{aligned}$$

Remove the brackets from a) $9(2x + 3y)$, b) $m(m + n)$.
a) The 9 must multiply both the term $2x$ and the term $3y$. Thus

Your solution

$9(2x + 3y) =$

$18x + 27y$

b) In the expression $m(m + n)$ the first m must multiply both terms in the brackets. Thus

Your solution

$m(m + n) =$

$m^2 + mn$

Example Remove the brackets from the expression $5x - (3x + 1)$ and simplify the result by collecting like terms.

Solution

The brackets in $-(3x + 1)$ were removed in an example on p.3.

$$\begin{aligned} 5x - (3x + 1) &= 5x - 1(3x + 1) \\ &= 5x - 3x - 1 \\ &= 2x - 1 \end{aligned}$$

Example Show that $\frac{-x-1}{4}$, $\frac{-(x+1)}{4}$ and $-\frac{x+1}{4}$ are all equivalent expressions.

Solution

Consider $-(x+1)$. Removing the brackets we obtain $-x-1$ and so

$$\frac{-x-1}{4} \quad \text{is equivalent to} \quad \frac{-(x+1)}{4}$$

A negative quantity divided by a positive quantity will be negative. Hence

$$\frac{-(x+1)}{4} \quad \text{is equivalent to} \quad -\frac{x+1}{4}$$

You should study all three expressions carefully to recognise the variety of equivalent ways in which we can write an algebraic expression.

Sometimes the bracketed expression can appear on the left, as in $(a+b)c$. To remove the brackets here we use the following rules:

Key Point

$$(a+b)c = ac + bc, \qquad (a-b)c = ac - bc$$

Note that when the brackets are removed both the terms in the brackets multiply c.

Example Remove the brackets from $(2x+3y)x$.

Solution

Both terms in the brackets multiply the x outside. Thus

$$\begin{aligned}(2x+3y)x &= 2x(x) + 3y(x)\\ &= 2x^2 + 3yx\end{aligned}$$

Remove the brackets from a) $(x+3)(-2)$, b) $(x-3)(-2)$.

Your solution

a) $(x+3)(-2) =$

a) Both terms in the bracket must multiply the -2. $-2x-6$

Your solution

b) $(x-3)(-2) =$

$-2x+6$

3. Removing brackets from expressions of the form $(a+b)(c+d)$

Sometimes it is necessary to consider two bracketed terms multiplied together. In the expression $(a+b)(c+d)$, by regarding the first bracket as a single term we can use the result in Section 1 to write it as $(a+b)c+(a+b)d$. Removing the brackets from each of these terms produces $ac+bc+ad+bd$. More concisely:

Key Point

$$(a+b)(c+d) = (a+b)c + (a+b)d = ac+bc+ad+bd$$

We see that each term in the first bracketed expression multiplies each term in the second bracketed expression.

Example Remove the brackets from $(3+x)(2+y)$

Solution

we find
$$\begin{aligned}(3+x)(2+y) &= (3+x)(2)+(3+x)y\\ &= (3)(2)+(x)(2)+(3)(y)+(x)(y) = 6+2x+3y+xy\end{aligned}$$

Example Remove the brackets from $(3x+4)(x+2)$ and simplify your result.

Solution

$$\begin{aligned}(3x+4)(x+2) &= (3x+4)(x) + (3x+4)(2)\\ &= 3x^2 + 4x + 6x + 8 = 3x^2 + 10x + 8\end{aligned}$$

Example Remove the brackets from $(a+b)^2$ and simplify your result.

Solution

When a quantity is squared it is multiplied by itself. Thus

$$\begin{aligned}(a+b)^2 = (a+b)(a+b) &= (a+b)a + (a+b)b\\ &= a^2 + ba + ab + b^2 = a^2 + 2ab + b^2\end{aligned}$$

Key Point

$(a+b)^2 = a^2 + 2ab + b^2$ $\qquad$ $(a-b)^2 = a^2 - 2ab + b^2$

Remove the brackets from the following expressions and simplify the results.

(a) $(x+7)(x+3)$, (b) $(x+3)(x-2)$,

Your solution

a) $(x+7)(x+3) =$

$x^2 + 7x + 3x + 21 = x^2 + 10x + 21$

Your solution

b) $(x+3)(x-2) =$

$x^2 + 3x - 2x - 6 = x^2 + x - 6$

Example Explain the distinction between $(x+3)(x+2)$ and $x+3(x+2)$.

Solution

In the first case removing the brackets we find

$$\begin{aligned}(x+3)(x+2) &= x^2+3x+2x+6\\ &= x^2+5x+6\end{aligned}$$

In the second case we have

$$x+3(x+2)=x+3x+6=4x+6$$

Note that in the second case the term $(x+2)$ is only multiplied by 3 and not by x.

Example Remove the brackets from $(s^2+2s+4)(s+3)$.

Solution

Each term in the first bracket must multiply each term in the second. Working through all combinations systematically we have

$$\begin{aligned}(s^2+2s+4)(s+3) &= (s^2+2s+4)(s) \quad + \quad (s^2+2s+4)(3)\\ &= s^3+2s^2+4s+3s^2+6s+12\\ &= s^3+5s^2+10s+12\end{aligned}$$

Exercises

1. Remove the brackets from each of the following expressions:

   a) $2(mn)$, b) $2(m+n)$, c) $a(mn)$, d) $a(m+n)$, e) $a(m-n)$,

   f) $(am)n$, g) $(a+m)n$, h) $(a-m)n$, i) $5(pq)$, j) $5(p+q)$,

   k) $5(p-q)$, l) $7(xy)$, m) $7(x+y)$, n) $7(x-y)$, o) $8(2p+q)$,

   p) $8(2pq)$, q) $8(2p-q)$, r) $5(p-3q)$, s) $5(p+3q)$ t) $5(3pq)$.

2. Remove the brackets from each of the following expressions:

   a) $4(a+b)$, b) $2(m-n)$, c) $9(x-y)$,

3. Remove the brackets from each of the following expressions and simplify where possible:

   a) $(2+a)(3+b)$, b) $(x+1)(x+2)$, c) $(x+3)(x+3)$, d) $(x+5)(x-3)$

4. Remove the brackets from each of the following expressions:

   a) $(7+x)(2+x)$, b) $(9+x)(2+x)$, c) $(x+9)(x-2)$, d) $(x+11)(x-7)$,

   e) $(x+2)x$, f) $(3x+1)x$, g) $(3x+1)(x+1)$, h) $(3x+1)(2x+1)$

   i) $(3x+5)(2x+7)$, j) $(3x+5)(2x-1)$, k) $(5-3x)(x+1)$ l) $(2-x)(1-x)$

5. Remove the brackets from $(s+1)(s+5)(s-3)$.

Answers

1. a) $2mn$, b) $2m+2n$, c) amn, d) $am+an$, e) $am-an$, f) amn, g) $an+mn$,
   h) $an-mn$, i) $5pq$, j) $5p+5q$, k) $5p-5q$, l) $7xy$, m) $7x+7y$, n) $7x-7y$,
   o) $16p+8q$, p) $16pq$, q) $16p-8q$, r) $5p-15q$, s) $5p+15q$, t) $15pq$
2. a) $4a+4b$, b) $2m-2n$, c) $9x-9y$
3. a) $6+3a+2b+ab$, b) x^2+3x+2, c) x^2+6x+9, d) $x^2+2x-15$
4. a) $14+9x+x^2$, b) $18+11x+x^2$, c) $x^2+7x-18$, d) $x^2+4x-77$, e) x^2+2x, f) $3x^2+x$,
   g) $3x^2+4x+1$
   h) $6x^2+5x+1$, i) $6x^2+31x+35$, j) $6x^2+7x-5$, k) $-3x^2+2x+5$, l) x^2-3x+2
5. $s^3+3s^2-13s-15$

4. Factorisation

A number is said to be **factorised** when it is written as a product. For example 21 can be factorised into 7×3. We say that 7 and 3 are **factors** of 21. Always remember that the factors of a number are *multiplied* together.

Algebraic expressions can also be factorised. Consider the expression $7(2x + 1)$. Removing the brackets we can rewrite this as

$$7(2x + 1) = 7(2x) + (7)(1) = 14x + 7.$$

Thus $14x + 7$ is equivalent to $7(2x + 1)$. We see that $14x + 7$ has factors 7 and $(2x + 1)$. The factors 7 and $(2x + 1)$ *multiply* together to give $14x + 7$. The process of writing an expression as a product of its factors is called **factorisation**. When asked to factorise $14x + 7$ we write

$$14x + 7 = 7(2x + 1)$$

and so we see that, here, factorisation can be regarded as reversing the process of removing brackets.

Always remember that the factors of an algebraic expression are *multiplied* together.

Example factorise the expression $4x + 20$.

Solution

Both terms in the expression $4x + 20$ are examined to see if they have any factors in common. Clearly 20 can be factorised as (4)(5) and so we can write

$$4x + 20 = 4x + (4)(5)$$

The factor 4 is common to both terms on the right; it is called a **common factor** and is placed at the front and outside brackets to give

$$4x + 20 = 4(x + 5)$$

Note that the factorised form can and should be checked by removing the brackets again.

Example factorise $z^2 - 5z$.

Solution

Note that since $z^2 = z \times z$ we can write

$$z^2 - 5z = z(z) - 5z$$

so that there is a common factor of z. Hence

$$z^2 - 5z = z(z) - 5z = z(z - 5)$$

Example factorise $6x - 9y$.

Solution

By observation we note that there is a common factor of 3. Thus

$$6x - 9y = 3(2x) - 3(3y) = 3(2x - 3y)$$

Identify the factor common to both $14z$ and $21w$. Hence factorise $14z + 21w$.

The factor common to both $14z$ and $21w$ is

Your solution

7

We can then write

Your solution

$14z + 21w =$

$7(2z + 3w)$

factorise $6x - 12xy$.

First identify any common factors. In this case there are two,

Your solution

6 and x

Then we can write

Your solution

$6x - 12xy =$

$6x(1 - 2y)$

If there is any doubt, check your answer by removing the brackets again.

Exercises

1. Factorise

   a) $5x + 15y$, b) $3x - 9y$, c) $2x + 12y$, d) $4x + 32z + 16y$, e) $\frac{1}{2}x + \frac{1}{4}y$.

   In each case check your answer by removing the brackets again.

2. Factorise

   a) $a^2 + 3ab$, b) $xy + xyz$, c) $9x^2 - 12x$

3. Explain why a is a factor of $a + ab$ but b is not. Factorise $a + ab$.

4. Explain why x^2 is a factor of $4x^2 + 3yx^3 + 5yx^4$ but y is not.

   Factorise $4x^2 + 3yx^3 + 5yx^4$.

Answers

1. a) $5(x + 3y)$, b) $3(x - 3y)$, c) $2(x + 6y)$, d) $4(x + 8z + 4y)$, e) $\frac{1}{2}(x + \frac{1}{2}y)$

2. a) $a(a + 3b)$, b) $xy(1 + z)$, c) $3x(3x - 4)$.

3. $a(1 + b)$. 4. $x^2(4 + 3yx + 5yx^2)$.

5. Factorising quadratic expressions

One of the most important forms, occurring in many areas of mathematics, physics and engineering, is the quadratic expression. Many quadratic expressions can be written as the product of two linear factors and, in this Section, we examine how these can be easily found.

Key Point

An expression of the form

$$ax^2 + bx + c$$

where a, b and c are numbers is called a **quadratic expression**.

The numbers b and c may be zero but a must not be zero (for, then, the quadratic reduces to a linear expression). The number a is called the **coefficient** of x^2, b is the coefficient of x and c is called the **constant term**.

Consider the product $(x+1)(x+2)$. Removing brackets yields $x^2 + 3x + 2$. Conversely, we see that the factors of $x^2 + 3x + 2$ are $(x+1)$ and $(x+2)$. However, if we were given the quadratic expression first, how would we factorise it ? The following examples show how to do this but note that not all quadratic expressions can be easily factorised.
To enable us to factorise a quadratic expression in which the coefficient of x^2 equals 1, we note the following expansion:

$$(x+m)(x+n) = x^2 + mx + nx + mn = x^2 + (m+n)x + mn$$

So, given a quadratic expression we can think of the coefficient of x as $m+n$ and the constant term as mn. Once the values of m and n have been found the factors can be easily obtained.

Example Factorise $x^2 + 4x - 5$.

Solution

Writing $x^2 + 4x - 5 = (x+m)(x+n) = x^2 + (m+n)x + mn$ we seek numbers m and n so that $m + n = 4$ and $mn = -5$. By trial and error it is not difficult to find that $m = 5$ and $n = -1$ (or, the other way round, $m = -1$ and $n = 5$). So we can write

$$x^2 + 4x - 5 = (x+5)(x-1)$$

The answer can be checked easily by removing brackets.

Factorise $x^2 + 6x + 8$.
The coefficient of x^2 is 1. We can write

$$x^2 + 6x + 8 = (x+m)(x+n) = x^2 + (m+n)x + mn$$

so that $m+n = 6$ and $mn = 8$. Try various possibilities for m and n until you find values which satisfy both of these equations.

Your solution

e.g. $m = 4$, $n = 2$ or, the other way round, $m = 2$, $n = 4$

Finally factorise the quadratic:

Your solution

$x^2 + 6x + 8 =$

$(x+4)(x+2)$

When the coefficient of x^2 is not equal to 1 it may be possible to extract a numerical factor. For example, note that $3x^2 + 18x + 24$ can be written as $3(x^2 + 6x + 8)$ and then factorised as in the previous example. Sometimes no numerical factor can be found and a slightly different approach may be taken. We will demonstrate a technique which can always be used to transform the given expression into one in which the coefficient of the squared variable equals 1.

Example Factorise $2x^2 + 5x + 3$.

Solution

First note the coefficient of x^2; in this case 2. Multiply the whole expression by this number and rearrange as follows:

$$2(2x^2 + 5x + 3) = 2(2x^2) + 2(5x) + 2(3) = (2x)^2 + 5(2x) + 6.$$

If we now introduce a new variable such that $z = 2x$ we find that the coefficient of the squared term equals 1. Thus we can write

$$(2x)^2 + 5(2x) + 6 \qquad \text{as} \qquad z^2 + 5z + 6$$

This can be factorised to give $(z + 3)(z + 2)$. Returning to the original variable by writing $z = 2x$ we find

$$2(2x^2 + 5x + 3) = (2x + 3)(2x + 2)$$

A factor of 2 can be extracted from the second bracket on the right so that

$$2(2x^2 + 5x + 3) = 2(2x + 3)(x + 1)$$

so that

$$2x^2 + 5x + 3 = (2x + 3)(x + 1)$$

As an alternative to the technique of the previous example, experience and practice can often help us to identify factors. For example suppose we wish to factorise $3x^2 + 7x + 2$. We write

$$3x^2 + 7x + 2 = (\qquad)(\qquad)$$

In order to obtain the term $3x^2$ we can place terms $3x$ and x in the brackets to give

$$3x^2 + 7x + 2 = (3x + \ ? \ )(x + \ ? \ )$$

In order to obtain the constant 2, we consider the factors of 2. These are 1,2 or -1,-2. By placing these factors in the brackets we can factorise the quadratic expression. Various possibilities exist: we could write $(3x + 2)(x + 1)$, $(3x + 1)(x + 2)$, $(3x - 2)(x - 1)$ or $(3x - 1)(x - 2)$, only one of which is correct. By removing brackets from each in turn we look for the factorisation which produces the correct middle term, $7x$. The correct factorisation is found to be

$$3x^2 + 7x + 2 = (3x + 1)(x + 2)$$

With practice you will be able to carry out this process quite easily.

factorise the quadratic expression

$$5x^2 - 7x - 6$$

Write

$$5x^2 - 7x - 6 = (\qquad\qquad)(\qquad\qquad)$$

To obtain the quadratic term $5x^2$, insert $5x$ and x in the brackets:

$$5x^2 - 7x - 6 = (5x + ?\ )(x + ?\ )$$

Now examine the factors of -6. These are

Your solution

$3, -2$ or $-3, 2$ or $-6, 1$ or $6, -1$

Use these factors to find which pair, if any, gives rise to the middle term, $-7x$, and complete the factorisation.

Your solution

$5x^2 - 7x - 6 =$

$(5x + 3)(x - 2)$

On occasions you will meet expressions of the form $x^2 - y^2$. Such an expression is known as the **difference of two squares**. Note that here we are finding the difference between two squared terms. It is easy to verify by removing brackets that this factorises as

$$x^2 - y^2 = (x + y)(x - y)$$

So, if you can learn to recognise such expressions it is an easy matter to factorise them.

Example Factorise

a) $x^2 - 36z^2$, b) $25x^2 - 9z^2$, c) $\alpha^2 - 1$

Solution

In each case we are required to find the difference of two squared terms.

(a) Note that $x^2 - 36z^2 = x^2 - (6z)^2$. This factorises as $(x + 6z)(x - 6z)$.

(b) Here $25x^2 - 9z^2 = (5x)^2 - (3z)^2$. This factorises as $(5x + 3z)(5x - 3z)$.

(c) $\alpha^2 - 1 = (\alpha + 1)(\alpha - 1)$.

Exercises

1. Factorise

   a) $x^2 + 8x + 7$, b) $x^2 + 6x - 7$, c) $x^2 + 7x + 10$, d) $x^2 - 6x + 9$, e) $x^2 + 5x + 6$.

2. Factorise

   a) $2x^2 + 3x + 1$, b) $2x^2 + 4x + 2$, c) $3x^2 - 3x - 6$, d) $5x^2 - 4x - 1$, e) $16x^2 - 1$,

   f) $-x^2 + 1$, g) $-2x^2 + x + 3$.

3. Factorise

   a) $x^2 + 9x + 14$, b) $x^2 + 11x + 18$, c) $x^2 + 7x - 18$, d) $x^2 + 4x - 77$, e) $x^2 + 2x$,

   f) $3x^2 + x$, g) $3x^2 + 4x + 1$, h) $6x^2 + 5x + 1$, i) $6x^2 + 31x + 35$, j) $6x^2 + 7x - 5$,

   k) $-3x^2 + 2x + 5$, l) $x^2 - 3x + 2$.

4. Factorise a) $z^2 - 144$, b) $z^2 - \frac{1}{4}$, c) $s^2 - \frac{1}{9}$

Answers

1. a) $(x+7)(x+1)$, b) $(x+7)(x-1)$, c) $(x+2)(x+5)$, d) $(x-3)(x-3)$, e) $(x+3)(x+2)$

2. a) $(2x+1)(x+1)$, b) $2(x+1)^2$, c) $3(x+1)(x-2)$, d) $(5x+1)(x-1)$,

   e) $(4x+1)(4x-1)$, f) $(x+1)(1-x)$, g) $(x+1)(3-2x)$

3. a) $(7+x)(2+x)$, b) $(9+x)(2+x)$, c) $(x+9)(x-2)$, d) $(x+11)(x-7)$,

   e) $(x+2)x$, f) $(3x+1)x$, g) $(3x+1)(x+1)$, h) $(3x+1)(2x+1)$

   i) $(3x+5)(2x+7)$, j) $(3x+5)(2x-1)$, k) $(5-3x)(x+1)$, l) $(2-x)(1-x)$

4. a) $(z+12)(z-12)$, b) $(z+\frac{1}{2})(z-\frac{1}{2})$, c) $(s+\frac{1}{3})(s-\frac{1}{3})$