Formulae and transposition

1.5

Introduction

Formulae are used frequently in almost all aspects of engineering in order to relate a physical quantity to one or more others. Many well-known physical laws are described using formulae. For example, you may have already seen Ohm's law, $v = iR$, or Newton's second law of motion, $F = ma$. In this section we describe the process of evaluating a formula, explain what is meant by the subject of a formula, and show how a formula is rearranged or transposed. These are basic skills required in all aspects of engineering.

Prerequisites

Before starting this Section you should . . .

① be able to add, subtract, multiply and divide algebraic fractions

Learning Outcomes

After completing this Section you should be able to . . .

✓ transpose a formula

1. Using formulae and substitution

In the study of engineering, physical quantities can be related to each other using a formula. The formula will contain variables and constants which represent the physical quantities. To evaluate a formula we must **substitute** numbers in place of the variables.

For example, Ohm's law provides a formula for relating the voltage, v, across a resistor with resistance value, R, to the current through it, i. The formula states

$$v = iR$$

We can use this formula to calculate v if we know values for i and R. For example, if $i = 13\,\text{A}$, and $R = 5\,\Omega$,then

$$\begin{aligned} v &= iR \\ &= (13)(5) \\ &= 65 \end{aligned}$$

The voltage is $65\,\text{V}$.

Note from this example that it is important to pay attention to the units of any physical quantities involved. Unless a consistent set of units is used a formula is not valid.

Example The kinetic energy, K, of an object of mass M moving with speed v can be calculated from the formula, $K = \frac{1}{2}Mv^2$.

Calculate the kinetic energy of an object of mass 5 kg moving with a speed of 2 ms^{-1}.

Solution

In this example $M = 5$ and $v = 2$. Substituting these values into the formula we find

$$\begin{aligned} K &= \frac{1}{2}Mv^2 \\ &= \frac{1}{2}(5)(2^2) \\ &= 10 \end{aligned}$$

In the SI system the unit of energy is the joule. Hence the kinetic energy of the object is 10 joules.

The area, A, of the circle of radius r can be calculated from the formula $A = \pi r^2$.

Equivalently, if we know the diameter of the circle, d, we can use the equivalent formula $A = \frac{\pi d^2}{4}$. Find the area of a circle having diameter 0.1m. Your calculator will be preprogrammed with the value of π.

The area of the circle is

Your solution

$A =$

$\frac{\pi(0.1)^2}{4} = 0.0079\text{m}^2$

Example The volume, V, of a circular cylinder is equal to its cross-sectional area, A, times its length, h.

Find the volume of a cylinder having diameter 0.1m and length 0.3m

Solution

We can use the result of the previous example to obtain the cross-sectional area. Then

$$\begin{aligned} V &= Ah \\ &= \frac{\pi(0.1)^2}{4} \times 0.3 \\ &= 0.0024 \end{aligned}$$

The volume is 0.0024m^3.

2. Rearranging a formula

In the formula for the area of a circle, $A = \pi r^2$, we say that A is the **subject** of the formula. A variable is the subject of the formula if it appears by itself on one side of the formula, usually the left-hand side, and nowhere else in the formula. If we are asked to **transpose** the formula for r, or **solve** for r, then we have to make r the subject of the formula. When transposing a formula *whaterver is done to one side is done to the other.* There are five rules that must be adhered to.

Key Point

You may carry out the following

- add the same quantity to both sides of the formula
- subtract the same quantity from both sides of the formula
- multiply both sides of the formula by the same quantity
- divide both sides of the formula by the same quantity
- take 'functions' of both sides of the formula: for example, square both sides, find the reciprocal of both sides.

Example Transpose the formula $p = 5t - 17$ for t.

Solution

We must try to obtain t on its own on the left-hand side. We do this in stages by using one or more of the five rules. For example by adding 17 to both sides of $p = 5t - 17$ we find

$$\begin{aligned} p + 17 &= 5t - 17 + 17 \\ \text{so that} \qquad p + 17 &= 5t \end{aligned}$$

Dividing both sides by 5 we obtain t on its own:

$$\frac{p + 17}{5} = t$$

so that $t = \frac{p+17}{5}$.

Example Transpose the formula $\sqrt{2q} = p$ for q.

Solution

First of all we square both sides to remove the square root around $2q$. Note that $(\sqrt{2q})^2 = 2q$. This gives

$$2q = p^2$$

Dividing both sides by 2 gives $q = \frac{p^2}{2}$.

Transpose the formula $v = \sqrt{t^2 + w}$ for w.

We must try to obtain w on its own on the left-hand side. We do this in several stages. First of all square both sides to remove the square root around $t^2 + w$. This gives

Your solution

$v^2 = t^2 + w$

Then subtract t^2 from both sides to obtain an expression for w.

Your solution

$v^2 - t^2 = w$

Finally, we can write down the formula for w:

Your solution

$w = v^2 - t^2$

Example Transpose $x = \frac{1}{y}$ for y.

Solution

We must try to obtain an expression for y. In the given formula y appears in the form of a fraction. Multiplying both sides by y has the effect of removing this fraction:

$$\begin{aligned}\text{multiply both sides of} \qquad x &= \frac{1}{y} \qquad \text{by } y \text{ to get}\\ yx &= y \times \frac{1}{y}\\ \text{so that} \qquad yx &= 1\end{aligned}$$

Dividing both sides by x leaves y on its own, $y = \frac{1}{x}$.

Alternatively: simply invert both sides.

Example Make R the subject of the formula

$$\frac{2}{R} = \frac{3}{x+y}$$

Solution

In the given form R appears in a fraction. Inverting both sides gives

$$\frac{R}{2} = \frac{x+y}{3}$$

Thus multiplying both sides by 2 implies

$$R = \frac{2(x+y)}{3}$$

Example Make R the subject of the formula $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$.

Solution

The two terms on the right can be added to give

$$\frac{1}{R_1} + \frac{1}{R_2} = \frac{R_2 + R_1}{R_1 R_2}$$

The given formula becomes

$$\frac{1}{R} = \frac{R_2 + R_1}{R_1 R_2}$$

Now inverting both sides gives

$$R = \frac{R_1 R_2}{R_2 + R_1}$$

Exercises

1. The formula for the volume of a cylinder is $V = \pi r^2 h$. Find V when $r = 5$cm and $h = 15$cm.

2. If $R = 5p^2$, find R when a) $p = 10$, b) $p = 16$.

3. For the following formulae, find y at the given values of x.

   (a) $y = 3x + 2$, $x = -1$, $x = 0$, $x = 1$.

   (b) $y = -4x + 7$, $x = -2$, $x = 0$, $x = 1$.

4. If $P = \frac{3}{QR}$ find P if $Q = 15$ and $R = 0.300$.

5. If $y = \sqrt{\frac{x}{z}}$ find y if $x = 13.200$ and $z = 15.600$.

6. Evaluate $M = \frac{\pi}{2r+s}$ when $r = 23.700$ and $s = -0.2$.

7. To convert a length measured in metres to one measured in centimetres, the length in metres is multiplied by 100. Convert the following lengths to cm. a) 5m, b) 0.5m, c) 56.2m

8. To convert an area measured in m^2 to one measured in cm^2, the area in m^2 is multiplied by 10^4. Convert the following areas to cm^2. a) $5m^2$, b) $0.33m^2$, c) $6.2m^2$

9. To convert a volume measured in m^3 to one measured in cm^3, the volume in m^3 is multiplied by 10^6. Convert the following volumes to cm^3. a) $15m^3$, b) $0.25m^3$, c) $8.2m^3$

10. If $\eta = \frac{4Q_P}{\pi d^2 Ln}$ evaluate η when $Q_P = 0.0003$, $d = 0.05$, $L = 0.1$ and $n = 2$.

11. For the following formulae, find y at the given values of x.

    (a) $y = 2 - x$, $x = -3$, $x = -1$, $x = 1$, $x = 2$.

    (b) $y = x^2$, $x = -2$, $x = -1$, $x = 0$, $x = 1$, $x = 2$.

12. The moment of inertia of an object is a measure of its resistance to rotation. It depends upon both the mass of the object and the distribution of mass about the axis of rotation. It can be shown that the moment of inertia, J, of a solid disc rotating about an axis through its centre and perpendicular to the plane of the disc, is given by the formula

$$J = \frac{1}{2}Ma^2$$

    where M is the mass of the disc and a is its radius. Find the moment of inertia of a disc of mass 12 kg and diameter 10 m. The SI unit of moment of inertia is kg m^2.

13. Transpose the given formulae to make the given variable the subject.

    a) $y = 3x - 7$, for x, b) $8y + 3x = 4$, for x, c) $8x + 3y = 4$ for y,

    d) $13 - 2x - 7y = 0$ for x.

14. Transpose the formula $PV = RT$ for a) V, b) P, c) R, d) T.

15. Transpose $v = \sqrt{x + 2y}$, a) for x, b) for y.

16. Transpose $8u + 4v - 3w = 17$ for each of u, v and w.

17. When a ball is dropped from rest onto a horizontal surface it will bounce before eventually coming to rest after a time T where

$$T = \frac{2v}{g}\left(\frac{1}{1-e}\right)$$

where v is the speed immediately after the first impact, and g is a constant called the acceleration due to gravity. Transpose this formula to make e, the coefficient of restitution, the subject.

18. Transpose $q = A_1\sqrt{\frac{2gh}{(A_1/A_2)^2-1}}$ for A_2.

19. Make x the subject of a) $y = \frac{r+x}{1-rx}$, b) $y = \sqrt{\frac{x-1}{x+1}}$.

Answers 1. 1178.1 cm^3

2. a) 500, b) 1280

3. a) $-1, 2, 5$, b) 15, 7, 3

4. P=0.667

5. $y = 0.920$

6. $M = 0.067$

7. a) 500 cm, b) 50 cm, c) 5620 cm.

8. a) 50000 cm^2, b) 3300 cm^2, c) 62000 cm^2.

9. a) 1500000 cm^3, b) 250000 cm^3, c) 8200000 cm^3.

10. $\eta = 0.764$. 11. a) 5,3,1,0, b) 4,1,0,1,4.

12. 150 kg m^2.

13. a) $x = \frac{y+7}{3}$, b) $x = \frac{4-8y}{3}$, c) $y = \frac{4-8x}{3}$, d) $x = \frac{13-7y}{2}$

14. a) $V = \frac{RT}{P}$, b) $P = \frac{RT}{V}$, c) $R = \frac{PV}{T}$, d) $T = \frac{PV}{R}$

15. a) $x = v^2 - 2y$, b) $y = \frac{v^2-x}{2}$

16. $u = \frac{17-4v+3w}{8}$, $v = \frac{17-8u+3w}{4}$, $w = \frac{8u+4v-17}{3}$

17. $e = 1 - \frac{2v}{gT}$

18. $A_2 = \mp\sqrt{\frac{A_1^2 q^2}{2A_1^2 gh + q^2}}$

19. a) $x = \frac{y-r}{1+yr}$, b) $x = \frac{1+y^2}{1-y^2}$