First Order ODEs

19.2

Introduction

Separation of variables is a technique commonly used to solve first-order ordinary differential equations. It is so-called because we rearrange the equation to be solved such that all terms involving the dependent variable appear on one side of the equation, and all terms involving the independent variable appear on the other. Integration completes the solution. Not all first-order equations can be rearranged in this way so this technique is not always appropriate. Further, it is not always possible to perform the integration even if the variables are separable. In this Section you will learn how to decide whether the method is appropriate, and how to apply it in such cases.

Then you will learn how to recognise when a first-order differential equation is an exact equation and how to solve an exact equation. An **exact** first-order differential equation is one which can be solved by simply integrating both sides. The left-hand side is expressed as the derivative of an expression in both the independent and dependent variables. Only very few first-order differential equations are exact. Some others may be converted simply to exact equations and that is also considered

An **exact** differential equation is one which can be solved by simply integrating both sides. Whilst such equations are few and far between an important class of differential equations can be converted into exact equations by multiplying through by a function known as the **integrating factor** for the equation. In the last part of this Section you will learn how to decide whether an equation is capable of being transformed into an exact equation, how to determine the integrating factor and how to obtain the solution of the original equation.

Prerequisites

Before starting this Section you should ...

① understand what is meant by a differential equation; (Section 19.1)

Learning Outcomes

After completing this Section you should be able to ...

✓ explain what is meant by *separating the variables* of a first-order differential equation

✓ determine whether a first-order differential equation is separable

✓ solve a variety of equations using this technique

1. What does it mean to say that we separate the variables ?

In this Section we consider differential equations which can be written in the form

$$\frac{\mathrm{d}y}{\mathrm{d}x} = f(x)g(y)$$

Note that the right-hand-side is a product of a function of x, and a function of y. Examples of such equations are

$$\frac{\mathrm{d}y}{\mathrm{d}x} = x^2 y^3, \qquad \frac{\mathrm{d}y}{\mathrm{d}x} = y^2 \sin x \quad \text{and} \quad \frac{\mathrm{d}y}{\mathrm{d}x} = y \ln x$$

Not all first-order equations can be written in this form. For example, it is not possible to rewrite the equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = x^2 + y^3$$

in the form

$$\frac{\mathrm{d}y}{\mathrm{d}x} = f(x)g(y)$$

Which of the following differential equations do you think can be written in the form

$$\frac{\mathrm{d}y}{\mathrm{d}x} = f(x)g(y) \ ?$$

If possible, rewrite each equation in this form.

$$\text{a)}\ \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{x^2}{y^2}, \qquad \text{b)}\ \frac{\mathrm{d}y}{\mathrm{d}x} = 4x^2 + 2y^2, \qquad \text{c)}\ y\frac{\mathrm{d}y}{\mathrm{d}x} + 3x = 7$$

Your solution

a) $\frac{\mathrm{d}y}{\mathrm{d}x} = x^2\left(\frac{1}{y^2}\right)$, b) cannot be written in the stated form, c) Write $y\frac{\mathrm{d}y}{\mathrm{d}x} = 7 - 3x$ so that $\frac{\mathrm{d}y}{\mathrm{d}x} = (7 - 3x) \times \frac{1}{y}$ which is in the required form.

The variables involved need not be x and y. Other equations of this class are

$$\frac{\mathrm{d}z}{\mathrm{d}t} = te^z \qquad \frac{\mathrm{d}\theta}{\mathrm{d}t} = -\theta \quad \text{and} \quad \frac{\mathrm{d}v}{\mathrm{d}r} = v\left(\frac{1}{r^2}\right)$$

Given a differential equation in the form

$$\frac{\mathrm{d}y}{\mathrm{d}x} = f(x)g(y)$$

we can divide through by $g(y)$ to obtain

$$\frac{1}{g(y)}\frac{\mathrm{d}y}{\mathrm{d}x} = f(x)$$

If we now integrate both sides of this equation with respect to x we obtain

$$\int \frac{1}{g(y)}\frac{\mathrm{d}y}{\mathrm{d}x}\mathrm{d}x = \int f(x)\mathrm{d}x$$

that is

$$\int \frac{1}{g(y)}\mathrm{d}y = \int f(x)\mathrm{d}x$$

We have *separated the variables* because the left-hand side contains only the variable y, and the right-hand side contains only the variable x. We can now try to integrate each side separately. If we can actually perform the required integrations we will obtain a relationship between y and x. Examples of this process are given in the next Section.

Key Point

The solution of the equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = f(x)g(y)$$

is found from

$$\int \frac{1}{g(y)}\mathrm{d}y = \int f(x)\mathrm{d}x$$

2. Applying the method of separation of variables

The method is illustrated in the following example.

Example Use the method of separation of variables to solve the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3x^2}{y}$$

(Note that this is sometimes written in the form $y\mathrm{d}y - 3x^2\mathrm{d}x = 0$.)

Solution

The equation already has the form

$$\frac{dy}{dx} = f(x)g(y)$$

where

$$f(x) = 3x^2 \qquad \text{and} \qquad g(y) = 1/y.$$

Dividing both sides by $g(y)$ we find

$$y\frac{dy}{dx} = 3x^2$$

Integrating both sides with respect to x gives

$$\int y\frac{dy}{dx}dx = \int 3x^2 dx$$

that is

$$\int y\, dy = \int 3x^2\, dx$$

Note that the left-hand side is an integral involving just y; the right-hand side is an integral involving just x. After integrating we find

$$\tfrac{1}{2}y^2 = x^3 + c$$

where c is a constant of integration. You might think that there would be a constant on the left-hand side too. You are quite right but the two constants can be combined into a single constant and so we need only write one. We now have a relationship between y and x as required. Often it is sufficient to leave your answer in this form but you may also be required to obtain an explicit relation for y in terms of x. In this particular case

$$y^2 = 2x^3 + 2c$$

so that

$$y = \pm\sqrt{2x^3 + D} \qquad \text{where } D = 2c$$

Use the method of separation of variables to solve the differential equation

$$\frac{dy}{dx} = \frac{\cos x}{\sin 2y} \qquad (\text{alternatively}: \quad \sin 2y\, dy - \cos x\, dx = 0)$$

Your solution

Separate the variables so that terms involving y and $\frac{dy}{dx}$ appear on the left, and terms involving just x appear on the right:

You should have obtained

$$\sin 2y \frac{dy}{dx} = \cos x$$

Your solution

Then integrate both sides with respect to x:

that is

$$\int \sin 2y \frac{dy}{dx}\,dx = \int \cos x\,dx$$

$$\int \sin 2y\,dy = \int \cos x\,dx$$

Your solution

Now integrate both sides:

You should have obtained

$$-\tfrac{1}{2}\cos 2y = \sin x + c$$

Your solution

Finally, rearrange to obtain an expression for y in terms of x:

$y = \frac{1}{2}\cos^{-1}(D - 2\sin x)$ where $D = -2c$

Exercises

1. Solve the equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{e}^{-x}}{y}.$$

2. Solve the equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2\mathrm{e}^{-y}$$

subject to the condition $y(0) = 1$.
3. Find the general solution of the following equations:

$$\text{a)}\ \frac{\mathrm{d}y}{\mathrm{d}x} = 3, \qquad \text{b)}\ \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{6\sin x}{y}$$

4. Find the general solution of the equation

$$\frac{\mathrm{d}x}{\mathrm{d}t} = t(x-2).$$

Find the particular solution which satisfies the condition $x(0) = 5$.
5. Some equations which do not appear to be separable can be made so by means of a suitable substitution. By means of the substitution $z = y/x$ solve the equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{y^2}{x^2} + \frac{y}{x} + 1$$

6. The equation

$$iR + L\frac{\mathrm{d}i}{\mathrm{d}t} = E$$

where R, L and E are constants arises in electrical circuit theory. This equation can be solved by separation of variables. Find the solution which satisfies the condition $i(0) = 0$.

Answers 1. $y = \pm\sqrt{D - 2\mathrm{e}^{-x}}$. 2. $y = \ln(x^3+e)$. 3a) $y = 3x+C$, b) $\frac{1}{2}y^2 = C-6\cos x$. 4. $x = 2+A\mathrm{e}^{t^2/2}$, $x = 2+3\mathrm{e}^{t^2/2}$. 5. $z = \tan(\ln Dx)$ so that $y = x\tan(\ln Dx)$. 6. $i = \frac{E}{R}(1-\mathrm{e}^{-t/\tau})$ where $\tau = L/R$.

3. What is an exact equation?

Consider the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2$$

By direct integration we find that the general solution of this equation is

$$y = x^3 + C$$

where C is, as usual, an arbitrary constant of integration.
Next, consider the differential equation

$$\frac{\mathrm{d}}{\mathrm{d}x}(y\ x) = 3x^2.$$

Again, by direct integration we find that the general solution is

$$y\ x = x^3 + C.$$

We now divide this equation by x to obtain

$$y = x^2 + \frac{C}{x}.$$

The differential equation $\frac{\mathrm{d}}{\mathrm{d}x}(y\ x) = 3x^2$ is called an **exact equation**. It can effectively be solved by integrating both sides.

Solve the equations (a) $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 5x^4$ (b) $\dfrac{\mathrm{d}}{\mathrm{d}x}(x^3 y) = 5x^4$

Your solution

(a) $y =$ (b) $y =$

a) $y = x^5 + C$ b) $x^3 y = x^5 + C$ so that $y = x^2 + \dfrac{C}{x^3}$.

If we consider examples of this kind in a more general setting we obtain the following Key Point:

Key Point

The solution of the equation

$$\frac{d}{\mathrm{d}x}(f(x) \cdot y) = g(x)$$

is

$$f(x) \cdot y = \int g(x)\,\mathrm{d}x \qquad \text{or} \qquad y = \frac{1}{f(x)} \int g(x)\,\mathrm{d}x$$

4. Solving exact equations

As we have seen, the differential equation $\frac{\mathrm{d}}{\mathrm{d}x}(y\ x) = 3x^2$ has solution $y = x^2 + C/x$. In the solution x^2 is called the definite part and C/x is called the indefinite part (containing the arbitrary constant of integration). If we take the definite part of this solution, i.e. $y_\mathrm{d} = x^2$, then

$$\frac{\mathrm{d}}{\mathrm{d}x}(y_\mathrm{d} \cdot x) = \frac{\mathrm{d}}{\mathrm{d}x}(x^2 \cdot x) = \frac{\mathrm{d}}{\mathrm{d}x}(x^3) = 3x^2.$$

Hence $y_\mathrm{d} = x^2$ is *a* solution of the differential equation.
Now if we take the indefinite part of the solution i.e. $y_\mathrm{i} = C/x$ then

$$\frac{d}{\mathrm{d}x}(y_\mathrm{i} \cdot x) = \frac{d}{\mathrm{d}x}\left(\frac{C}{x} \cdot x\right) = \frac{d}{\mathrm{d}x}(C) = 0.$$

It is always the case that the general solution of an exact equation is in two parts: a definite part $y_\mathrm{d}(x)$ which is *a* solution of the differential equation and an indefinite part $y_\mathrm{i}(x)$ which satisfies a simpler version of the differential equation in which the right-hand side is zero.

Solve the equation

$$\frac{\mathrm{d}}{\mathrm{d}x}(y\cos x) = \cos x$$

Verify that the indefinite part of the solution satisfies the equation

$$\frac{\mathrm{d}}{\mathrm{d}x}(y\cos x) = 0.$$

Your solution

(First part: hint —*integrate both sides of the differential equation*).

You should obtain $y = \tan x + C\sec x$ since, by integrating both sides of the differential equation:

$$y\cos x = \int \cos x\,\mathrm{d}x = \sin x + C \qquad \text{leading to} \qquad y = \tan x + C\sec x$$

Your solution

Second part: hint — *the indefinite part contains the arbitrary constant*

Now the indefinite part of the solution is $y_i = C\sec x$ (the part containing the arbitrary constant) and so $y_i \cos x = C$ and

$$\frac{\mathrm{d}}{\mathrm{d}x}(y_i \cos x) = \frac{d}{dx}(C) = 0$$

5. Recognising an exact equation

The equation $\frac{d}{\mathrm{d}x}(y\ x) = 3x^2$ is exact, as we have seen. If we expand the left-hand side of this equation (i.e. differentiating a product) we obtain

$$x\frac{\mathrm{d}y}{\mathrm{d}x} + y.$$

Hence the equation

$$x\frac{\mathrm{d}y}{\mathrm{d}x} + y = 3x^2$$

is exact, but it is not so obvious that it is exact as for the original form. This leads to our second Key Point.

Key Point

The equation

$$f(x)\frac{\mathrm{d}y}{\mathrm{d}x} + y\ f'(x) = g(x)$$

is exact. It can be re-written as

$$\frac{d}{\mathrm{d}x}(y\ f(x)) = g(x) \qquad \text{so that} \qquad y\ f(x) = \int g(x)\ \mathrm{d}x$$

Example Solve the equation

$$x^3\frac{\mathrm{d}y}{\mathrm{d}x} + 3x^2y = x$$

Solution

Comparing this equation with the form in the previous Key Point we see that $f(x) = x^3$ and $g(x) = x$. Hence the equation can be written

$$\frac{\mathrm{d}}{\mathrm{d}x}(y\ x^3) = x$$

which has solution

$$y\ x^3 = \int x\,\mathrm{d}x = \tfrac{1}{2}x^2 + C.$$

Therefore

$$y = \frac{1}{2x} + \frac{C}{x^3}.$$

Solve the equation $\sin x \frac{\mathrm{d}y}{\mathrm{d}x} + y\cos x = \cos x$

Your solution

You should obtain $y = 1 + C\mathrm{cosec}\,x$ since, here $f(x) = \sin x$ and $g(x) = \cos x$. Then

$$\frac{\mathrm{d}}{\mathrm{d}x}(y\sin x) = \cos x \qquad \text{and} \qquad y\sin x = \int \cos x\,\mathrm{d}x = \sin x + C$$

Finally $y = 1 + C\ \mathrm{cosec}\ x$

Exercises

1. Solve the equation $\frac{\mathrm{d}}{\mathrm{d}x}(y\ x^2) = x^3$.

2. Solve the equation $\frac{\mathrm{d}}{\mathrm{d}x}(y\ \mathrm{e}^x) = \mathrm{e}^{2x}$ given the condition $y(0) = 2$.

3. Solve the equation $\mathrm{e}^{2x}\frac{\mathrm{d}y}{\mathrm{d}x} + 2\mathrm{e}^{2x}y = x^2$.

4. Show that the equation $x^2\frac{\mathrm{d}y}{\mathrm{d}x} + 2x\ y = x^3$ is exact and obtain its solution.

5. Show that the equation $x^2\frac{\mathrm{d}y}{\mathrm{d}x} + 3x\ y = x^3$ is not exact. Multiply the equation by x and show that the resulting equation *is* exact and obtain its solution.

Answers 1. $y = \dfrac{x^2}{4} + \dfrac{C}{x^2}$. 2. $y = \frac{1}{2}e^{x} + \frac{2}{3}e^{-x}$. 3. $y = \left(\frac{1}{3}x^3 + C\right)e^{-2x}$. 4. $y = \dfrac{1}{4}x^2 + \dfrac{C}{x^2}$.

5. $y = \dfrac{1}{5}x^2 + \dfrac{C}{x^3}$.

6. How does an integrating factor work?

The equation

$$x^2\frac{dy}{dx} + 3x\ y = x^3$$

is not exact. However, if we multiply it by x we obtain the equation

$$x^3\frac{dy}{dx} + 3x^2 y = x^4.$$

This can be re-written as

$$\frac{d}{dx}(x^3 y) = x^4$$

which is an exact equation with solution

$$x^3 y = \int x^4 dx = \frac{1}{5}x^5 + C$$

and hence

$$y = \frac{1}{5}x^2 + \frac{C}{x^3}.$$

The function by which we multiplied the given differential equation in order to make it exact is called an integrating factor. In this example the integrating factor is simply x itself.

Which of the following differential equations can be made exact by multiplying by x^2?

(a) $\dfrac{dy}{dx} + \dfrac{2}{x}y = 4$ (b) $x\dfrac{dy}{dx} + 3y = x^2$ (c) $\dfrac{1}{x}\dfrac{dy}{dx} - \dfrac{1}{x^2}y = x$

(d) $\dfrac{1}{x}\dfrac{dy}{dx} + \dfrac{1}{x^2}y = 3.$

Where possible, write the exact equation in the form $\dfrac{d}{dx}(f(x)\ y) = g(x)$.

Your solution

(a) Yes. $x^2\frac{dy}{dx} + 2x\,y = 4x^2$ becomes $\frac{d}{dx}(x^2y) = 4x^2$.

(b) Yes. $x^3\frac{dy}{dx} + 3x^2y = x^4$ becomes $\frac{d}{dx}(x^3y) = x^4$.

(c) This equation is already exact as it can be written in the form $\frac{d}{dx}\left(\frac{1}{x}\,y\right) = x$.

(d) Yes. $x\,\frac{dy}{dx} + y = 3x^2$ becomes $\frac{d}{dx}(xy) = 3x^2$.

7. Finding the integrating factor for linear differential equations

The differential equation governing the current i in a circuit with inductance L and resistance R in series subject to a constant applied electromotive force $E\cos\omega t$ where E and ω are constants is

$$L\,\frac{\mathrm{d}i}{\mathrm{d}t} + Ri = E\cos\omega t \qquad (1)$$

This is an example of a linear differential equation in which i is the dependent variable and t is the independent variable. The general standard form of a linear first-order differential equation is normally written with 'y' as the dependent variable and with 'x' as the independent variable and arranged so that the coefficient of $\frac{\mathrm{d}y}{\mathrm{d}x}$ is 1. That is, it takes the form:

$$\frac{\mathrm{d}y}{\mathrm{d}x} + f(x)\;y = g(x)$$

in which $f(x)$ and $g(x)$ are functions of x.
In our example x is replaced by t and y by i to produce $\frac{\mathrm{d}i}{\mathrm{d}t} + f(t)\;i = g(t)$. The function $f(t)$ is the coefficient of the dependent variable in the differential equation. We shall describe the method of finding the integrating factor for (1) and then generalise it to a linear differential equation written in standard form.

Step 1 Write the differential equation in standard form i.e. with the coefficient of the derivative equal to 1. Here we need to divide through by L:

$$\frac{\mathrm{d}i}{\mathrm{d}t} + \frac{R}{L}i = \frac{E}{L}\cos\omega t.$$

Step 2 Integrate the coefficient of the dependent variable (that is, $f(t) \equiv R/L$) with respect to the independent variable (that is, t), and ignoring the constant of integration

$$\int \frac{R}{L}\,\mathrm{d}t = \frac{R}{L}\,t.$$

Step 3 Take the exponential of the function obtained in step 2.

This is the integrating factor (I.F.)

$$\text{I.F.} = \mathrm{e}^{Rt/L}.$$

This leads to the following Key Point on integrating factors:

Key Point

The linear differential equation (written in standard form):

$$\frac{\mathrm{d}y}{\mathrm{d}x} + f(x)y = g(x) \qquad \text{has an integrating factor} \qquad \text{I.F.} = \exp\left[\int f(x)\mathrm{d}x\right]$$

Find the integrating factor for the equations

(a) $x\,\dfrac{\mathrm{d}y}{\mathrm{d}x} + 2x\,y = x\mathrm{e}^{-2x}$ (b) $t\,\dfrac{\mathrm{d}i}{\mathrm{d}t} + 2t\,i = t\mathrm{e}^{-2t}$ (c) $\dfrac{\mathrm{d}y}{\mathrm{d}x} - (\tan x)y = 1.$

Your solution

(a) **Step 1** Divide by x to obtain $\dfrac{dy}{dx} + 2y = e^{-2x}$

Step 2 The coefficient of the independent variable is 2 hence $\int 2\ dx = 2x$

Step 3 $I.F. = e^{2x}$

(b) The only difference from (a) is that i replaces y and t replaces x. Hence $I.F. = e^{2t}$.

(c) **Step 1** This is already in the standard form.

Step 2 $\int -\tan x\ dx = \int \dfrac{-\sin x}{\cos x}\ dx = \ln\cos x$.

Step 3 $I.F. = e^{\ln\cos x} = \cos x$

8. Solving equations via the Integrating Factor

Having found the integrating factor for a linear equation we now proceed to the solution of the equation.
Returning to the differential equation, written in standard form:

$$\frac{di}{dt} + \frac{R}{L}i = \frac{E}{L}\cos\omega t$$

for which the integrating factor is

$$e^{Rt/L}$$

we multiply the equation by the integrating factor to obtain

$$e^{Rt/L}\,\frac{di}{dt} + \frac{R}{L}\,e^{Rt/L}\ i = \frac{E}{L}\,e^{Rt/L}\cos\omega t$$

At this stage you will find the left-hand side of this equation can *always* be simplified:

$$\frac{d}{dt}(e^{Rt/L}\ i) = \frac{E}{L}\,e^{Rt/L}\cos\omega t.$$

Now this is in the form of an exact differential equation and so we can simply integrate both sides to obtain the solution:

$$e^{Rt/L}\ i = \frac{E}{L}\int e^{Rt/L}\cos\omega t\ dt.$$

All that remains is to complete the integral on the right-hand side. Using the method of integration by parts we find

$$\int e^{Rt/L}\cos\omega t\ dt = \frac{L}{L^2\omega^2 + R^2}\left[\omega L\ \sin\omega t + R\cos\omega t\right]e^{Rt/L}$$

Hence

$$e^{Rt/L}\ i = \frac{E}{L^2\omega^2 + R^2}\left[\omega L\sin\omega t + R\cos\omega t\right]\ e^{Rt/L} + C.$$

Finally

$$i = \frac{E}{L^2\omega^2 + R^2}\left[\omega L\sin\omega t + R\cos\omega t\right] + C\ e^{-Rt/L}.$$

is the solution to the original differential equation (1). Note that, as we should expect for the solution to a first-order differential equation, it contains a single arbitrary constant C.

Using the integrating factors found earlier find the general solutions to the differential equations

(a) $x^2\dfrac{\mathrm{d}y}{\mathrm{d}x} + 2x^2y = x^2\mathrm{e}^{-2x}$ (b) $t^2\dfrac{\mathrm{d}y}{\mathrm{d}t} + 2t^2y = t^2\mathrm{e}^{-2t}$ (c) $\dfrac{\mathrm{d}y}{\mathrm{d}x} - (\tan x)y = 1$.

Your solution

(a) The standard form is $\dfrac{\mathrm{d}y}{\mathrm{d}x} + 2y = \mathrm{e}^{-2x}$ for which the integrating factor is e^{2x}. Then, on multiplying through by e^{2x} we have

$$
\begin{aligned}
\mathrm{e}^{2x}\frac{\mathrm{d}y}{\mathrm{d}x} + 2\mathrm{e}^{2x}y &= 1 \\
\text{i.e.}\quad \frac{d}{\mathrm{d}x}\left(\mathrm{e}^{2x}y\right) &= 1 \quad\text{so that}\quad \mathrm{e}^{2x}y = x + C \\
\text{leading to}\quad y &= (x + C)\mathrm{e}^{-2x}
\end{aligned}
$$

(b) The general solution is $y = (t + C)\mathrm{e}^{-2t}$

(c) The equation is in standard form and the integrating factor is $\cos x$.

$$
\begin{aligned}
\text{then}\quad \frac{d}{\mathrm{d}x}(\cos x\ y) &= \cos x \quad\text{so that}\quad \cos x\ y = \int \cos x\mathrm{d}x = \sin x + C \\
\text{giving}\quad y &= \tan x + C\sec x
\end{aligned}
$$

Exercises

1. Solve the equation $x^2\dfrac{dy}{dx} + x\ y = 1$.

2. Find the solution of the equation $x\dfrac{dy}{dx} - y = x$ subject to the condition $y(1) = 2$.

3. Find the general solution of the equation $\dfrac{dy}{dt} + (\tan t)\ y = \cos t$.

4. Solve the equation $\dfrac{dy}{dt} + (\cot t)\ y = \sin t$.

5. The temperature θ (measured in degrees) of a body immersed in an atmosphere of varying temperature is given by $\dfrac{d\theta}{dt} + 0.1\theta = 5 - 2.5t$. Find the temperature at time t if $\theta = 60°$ when $t = 0$.

6. In an LR circuit with applied voltage $E = 10(1 - e^{-0.1t})$ the current i is given by

$$L\frac{di}{dt} + Ri = 10(1 - e^{-0.1t}).$$

If the initial current is i_0 find i subsequently.

Answers 1. $y = \dfrac{1}{x}\ln x + \dfrac{C}{x}$.

2. $y = x\ln x + 2x$ 3. $y = (t + C)\cos t$

4. $y = \left(\frac{1}{2}t - \frac{1}{4}\sin 2t + C\right)\operatorname{cosec} t$ 5. $\theta = 300 - 25t - 240e^{-0.1t}$

6. $i = \dfrac{10}{R} - \left(\dfrac{100}{10R - L}\right)e^{-0.1t} + \left[i_0 + \dfrac{10L}{R(10R - L)}\right]e^{-Rt/L}$.