Modelling Exercises

6.6

Introduction

Engineering examples and exercises employing exponential functions and logarithmic functionss.

Prerequisites

Before starting this Section you should ...

① have knowledge logarithms to base 10

② be able to solve equations involving logarithms and exponentials

③ be familiar with the laws of logarithms

Learning Outcomes

After completing this Section you should be able to ...

✓

✓

1. Modelling Exercises

Exponential increase

(a) Look back at Section 2 of Workbook 6 to ascertain the definitions of *an* exponential function and *the* exponential function.

(b) List examples from this Workbook of contexts in which exponential functions are appropriate.

Your solution

(a) *An* exponential function has the form $y = a^x$ where $a > 0$. *The* exponential function has the form $y = e^x$ where $e = 2.718282......$

(b) It is stated that exponential functions are useful when modelling the shape of a hanging chain or rope under the effect of gravity or for modelling exponential growth or decay.

Let's look at a specific example of the exponential function used to model a population increase, i.e.

$$P = 12e^{0.01t} \qquad (0 \leq t \leq 100)$$

where P is the number in the population in millions at time t in years.

(a) What does this function imply about the population when $t = 0$?

(b) What is the range of validity?

(c) What does the function imply about values of P for $t > 0$?

Your solution

(a) At $t = 0$, $P = 12$ which represents the initial population. (Recall that $e^0 = 1$).

(b) The time interval during which the model is valid is stated as $(0 \leq t \leq 10)$ so the model is valid until the end of the tenth year after the start of timing.

(c) Since for $t > 0$, $e^{0.01t} > 1$, this implies that $P > 12$. As t increases towards 10, P becomes very large, i.e. greater than the current population of the earth!

Note that exponential population growth of the form $P = P_0 e^{kt}$ means that as t becomes large and positive, P becomes very large. Normally such a population model would be used to predict values of P for $t > 0$, where $t = 0$ represents the present or some fixed time when the population is known. In Figure 6.1, values of P are shown for $t < 0$. These correspond to extrapolation of the model into the past. Note that as t becomes increasingly negative, P becomes very small but is never zero or negative. Indeed e^{kx} is positive for any value of x. The parameter k is called the *instantaneous fractional growth rate*.

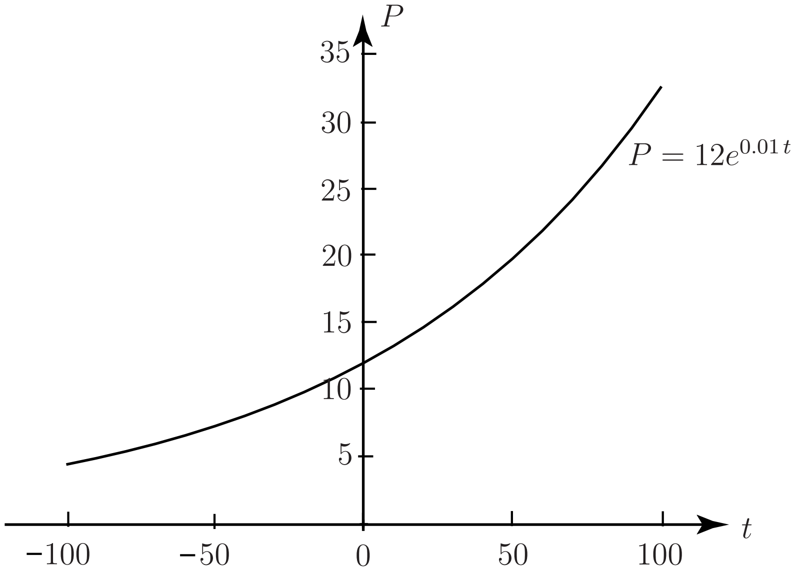


Figure 6.1 The function $P = 12e^{0.01t}$

In the previous example of exponential population growth, if k is doubled from 0.01 to 0.02, while keeping the initial population constant, then the appropriate function is

$$P = 12e^{0.02t}$$

This case, with k=0.02, implies a faster growth for $t > 0$. This is clear in the graphs for $k = 0.01$ and $k = 0.02$ in Figure 6.2. The functions are plotted up to 200 to emphasize the increasing difference as t increases.

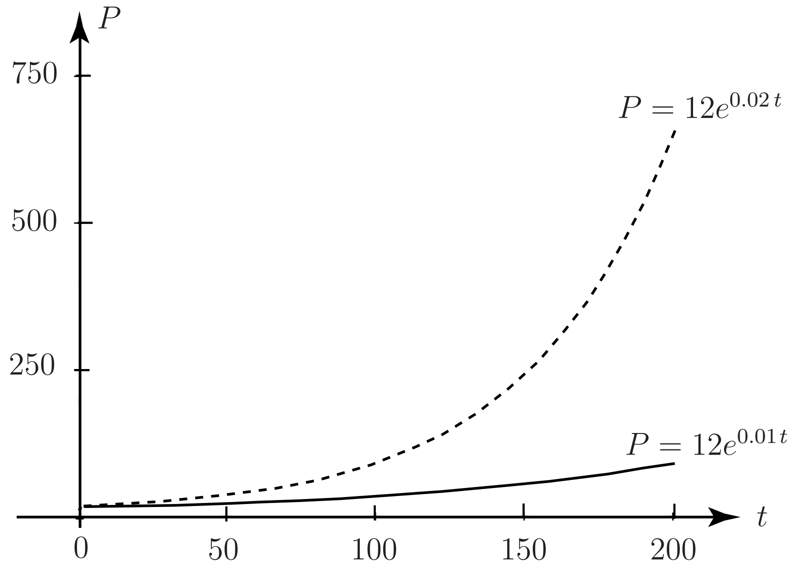


Figure 6.2 Comparison of the functions $P = 12e^{0.01t}$ and $P = 12000000e^{0.02t}$

The exponential function may be used in models for other types of growth as well as population growth. A general form may be written

$$y = ae^{bx} \qquad (c \leq x \leq d)$$

where a represents the value of y at $x = 0$. The value a is the intercept on the y-axis of a graphical representation of the function. The value b controls the rate of growth and c and d represent limits on x.

In the general form, a and b represent the *parameters* of the exponential function which can be selected to fit any given modelling situation where an exponential function might be an appropriate choice.

Linearisation of exponential functions

This sub-section relates to the description of log-linear plots in Workbook 6. Frequently in engineering, the question arises of how the parameters of an exponential function might be found from given data. The method follows from the fact that it is possible to undo the exponential function and obtain a linear function by means of the logarithmic functions. Before showing the implications of this method, it may be necessary to remind you of some rules for manipulating exponentials and logarithms. These are summarised in Tables 6.1 and 6.2.

number	rule
1	$e^x \times e^y = e^{x+y}$
2	$e^x / e^y = e^{x-y}$
3	$(e^x)^y = e^{xy}$
4	$e^0 = 1$
5	$e^1 = e$
6	$e^{\ln(x)} = \ln(e^x) = x$
7	$A^x = e^{kx}$ where $e^k = A$ (so $k = \ln(A)$)

Table 6.1 Rules for manipulating exponentials

number	rule
1	$\ln(x) + \ln(y) = \ln(xy) \qquad x, y > 0$
2	$\ln(x) - \ln(y) = \ln(x/y) \qquad x, y > 0$
3	$\ln(x^y) = y\ln(x) \quad x > 0$ and y is a real number
4	$\ln 1 = 0$

Table 6.2 Rules for manipulating logarithms

Let's try undoing the exponential in the particular example

$$P = 12e^{0.01t}$$

We take the natural logarithm of both sides. The *natural logarithm* of P is written $\ln(P)$ which means logarithm to the base e. Similarly, the natural logarithm of the right-hand side is represented by writing ln in front of a bracket containing the whole of the right hand side. So

$$\ln(P) = \ln(12e^{0.01t})$$

The result of using Rule 1 in Table 6.2 is

$$\ln(P) = \ln(12) + \ln(e^{0.01t}).$$

The natural logarithmic functions *undoes* the exponential function, so by Rule 7 of Table 6.1,

$$\ln(e^{0.01t}) = 0.01t$$

and the original equation for P is turned into

$$\ln(P) = \ln(12) + 0.01t.$$

Compare this with the general form of linear function $y = ax + b$ (Workbook 2).

$$\begin{array}{lll} y = & ax & +b \\ \downarrow & \downarrow & \downarrow \\ \ln(P) = & 0.01t & + \ln(12) \end{array}$$

If we regard $\ln(P)$ as equivalent to y, 0.01 as equivalent to a, t as equivalent to x, and $\ln(12)$ as equivalent to b, then we can identify a linear relationship between $\ln(P)$ and t. A plot of $\ln(P)$ against t should result in a straight line, of slope 0.01, which crosses the $\ln(P)$ axis at $\ln(12)$. (Such a plot is called a *log-lin* plot.) This is not particularly interesting here because we know the values 12 and 0.01 already. Suppose, though, we want to try using the general form of the exponential function

$$P = ae^{bt} \qquad (c \le t \le d)$$

to create a continuous model for a population for which we have some data. The first thing to do is to take logarithms of both sides

$$\ln(P) = \ln(ae^{bt}) \qquad (c \le t \le d).$$

Rule 1 from Table 6.2 then gives

$$\ln(P) = \ln(a) + \ln(e^{bt}) \qquad (c \le t \le d).$$

But, by Rule 7 from Table 6.1, $\ln(e^{bt}) = bt$, so this means that

$$\ln(P) = \ln(a) + bt \qquad (c \le t \le d).$$

So, given some population versus time data, for which you suspect some version of the exponential function to be appropriate, plot the natural logarithm of population against time. If the exponential function is appropriate, the resulting data points should lie on or near a straight line. The slope of the straight line will give a value for b and the intercept with the $\ln(P)$ axis will give a value for $\ln(a)$. You will have carried out a *logarithmic transformation* of the original data for P. We say the original variation has been *linearised*. A similar procedure will work also if the exponential function that is used to create the model is any exponential function rather than the natural exponential function. For example, suppose that we try to use the function

$$P = A \times 2^{Bt} \qquad (C \leq t \leq D),$$

where A and B are constant parameters to be derived from the given data. We can take natural logarithms again to give

$$\ln(P) = \ln(A \times 2^{B}) \qquad (C \leq t \leq D).$$

Rule 1 from Table 6.2 then gives

$$\ln(P) = \ln(A) + \ln(2^{Bt}) \qquad (C \leq t \leq D).$$

Rule 3 from Table 6.2 means that the logarithm of 2 raised to some power is equal to the product of that power and the logarithm of 2 so

$$\ln(2^{Bt}) = Bt\ln(2) = B\ln(2)\ t$$

and so

$$\ln(P) = \ln(A) + B\ln(2)\ t \qquad (C \leq t \leq D).$$

Again we have a straight line function with the same intercept as before but with slope $B\ln(2)$.

The amount of money £M owed after earning interest of 5% p.a. for N years is worked out as

$$M = 1.05^N$$

Find a linearised form of this equation.

Your solution

Take natural logarithms of both sides.

$$\ln(M) = \ln(1.05^N).$$

Rule 3 of Table 3.2 gives

$$\ln(M) = N\ln(1.05).$$

So a plot of $\ln(M)$ against N would be a straight line passing through (0,0) with slope $\ln(1.05)$.

The linearisation procedure also works if logarithms other than natural logarithms are used. Let's start again with

$$P = A \times 2^{Bt} \qquad (C \le t \le D),$$

If we take logarithms to base 10 instead of natural logarithms we get

$$\log_{10}(P) = \log_{10}(A \times 2^{B}) \qquad (C \le t \le D).$$

Rule 1 from Table 6.2 then gives

$$\log_{10}(P) = \log_{10}(A) + \log_{10}(2^{Bt}) \qquad (C \le t \le D).$$

Use of Rule 3 from Table 6.2 gives the result

$$\log_{10}(P) = log_{10}(A) + B\log_{10}(2)\ t \qquad (C \le t \le D).$$

(a) Write down the straight line function corresponding to taking logarithms to the base 10 of the general exponential function

$$P = ae^{bt} \qquad (c \le t \le d)$$

(b) Write down the slope of this line.

Your solution

(a) $\log_{10}(P) = \log_{10}(a) + (b\log_{10}(e))t \qquad (c \le t \le d)$

(b) $b\ log_{10}(e)$

It is not always necessary to declare the subscript 10 when indicating logarithms to base 10. It has been done here to make the distinction from natural logarithms clearer. If you met the abbreviation 'log' for the logarithmic functions elsewhere, it may imply "to the base 10". In the remainder of this chapter, the subscript 10 is dropped where $\log_{10}$ is implied.

2. Exponential Decrease

Consider the depreciation costs per year, £D, for a car in terms of the age A years of the car. The car was bought for £10500. The function

$$D = 10500e^{(-0.25A)} \qquad (0 \le A \le 6)$$

could be considered appropriate on the ground that (a) D had a fixed value of £10500 when

$A = 0$, (b) D decreases as A increases and (c) D decreases faster when A is small than when A is large. A plot of this function is shown in Figure 6.3.

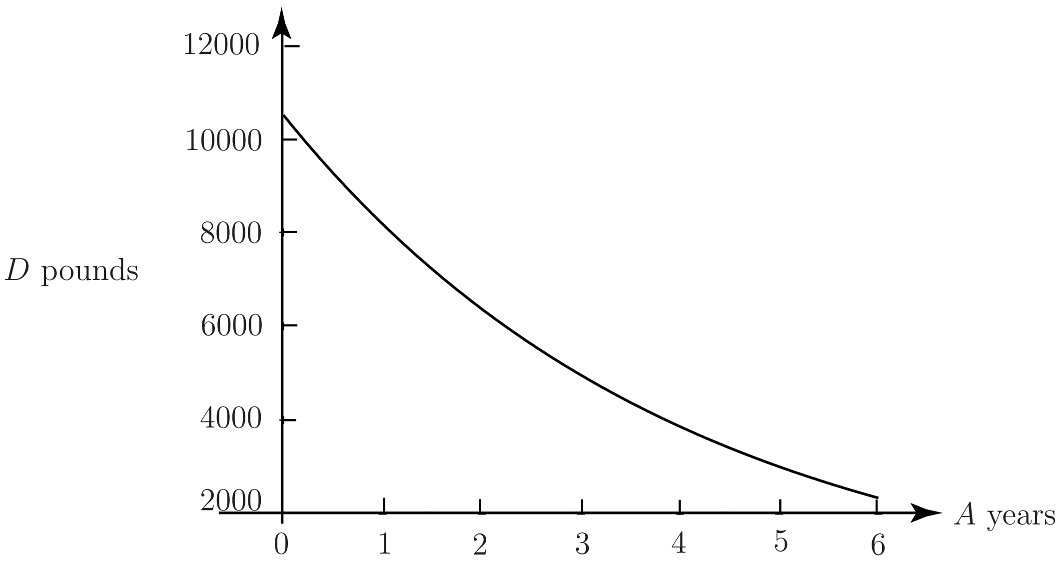


Figure 6.3 Plot of car depreciation over 6 years

A different sort of decreasing exponential function arises in the context of mortgage repayments. If the mortgage is represented by M in pounds and time by t in years then a specific example is

$$M = 15000 - 5000(1.05)^t \qquad (0 \le t \le 23).$$

When $t = 0$,

$$M = 15000 - 5000e^0 = 15000 - 5000 = 10000$$

which corresponds to the initial debt of £10000.

5% corresponds to 0.05 and $10000(1.05)^{23}$ represents the total amount paid over 23 years. This total amount divided by 23 represents the annual instalment. So this equation corresponds to a loan of a £10000 at a fixed rate of interest of 5% per annum being repaid over 23 years by annual instalments of £1335, $(1.05)^t$ is an exponential function. It can be replaced by a natural exponential function so that the equation for M has the form

$$M = 15000 - 5000e^{kt} \qquad (0 \le t \le 23),$$

where k is a positive constant. Comparing these two formulae we have:

$$1.05^t = e^{kt}$$

so, taking logs of both sides:

$$\ln(1.05) = k.$$

This gives a value for k of $0.0487901642 \approx 0.049$ to 3 decimal places.

As t increases, this new form for M is such that an exponentially-increasing term ($5000e^{kt}$) is being subtracted from a constant (15000). So we expect M to decrease at an increasing rate.

In the context of mortgage repayments, we want to know when $M = 0$ since that represents the time at which the mortgage is repaid, and for the given example this will be some time during the 23rd year.

a) Check whether the value of k worked out earlier is consistent with the amount owed being paid off during the 23rd year.

(b) Start with a more general form, $M = A - Be^{kt} \qquad (0 \leq t \leq T)$.

(i) Find the value of M at $t = 0$.
(ii) Find the value of k so that $M = 0$ when $t = T$.
(iii) Sketch a graph of M against t.

Your solution

(a) When $t = 23$,

$$15000 - 5000e^{23k} = 15000 - 5000e^{23\times 0.0487901642}$$

$$= 15000 - 15357.62$$

$$= -357.62.$$

This means that by the end of the 23rd year there would have been an overpayment of £357.62. A similar calculation with $t = 22$ produces an amount owing of just over £373. This is consistent with the amount owing becoming zero some time during the 23rd year.

(b) (i) When $t = 0$, $M = A - Be^0 = A - B$

(ii) When $t = T$, M = 0 so $0 = A - Be^{kT}$

$$\therefore \quad e^{kT} = \frac{A}{B} \quad \text{or} \quad k = \frac{1}{T}\ln(\frac{A}{B})$$

(iii) See Figure 6.4.

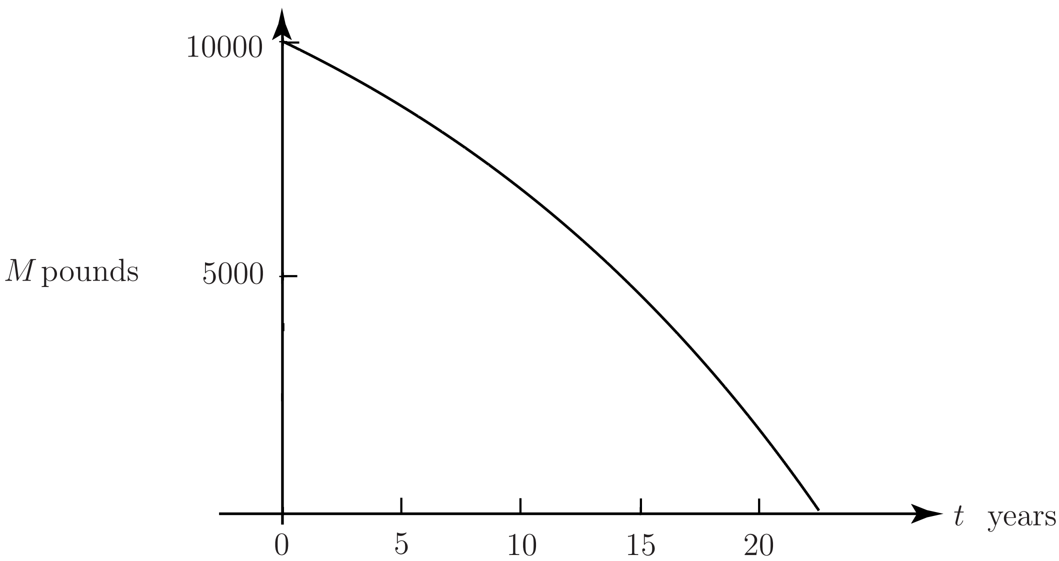


Figure 6.4 Graph of M against t

Note how the slope of the curve becomes more negative as t increases. Your work for part (b) of the last exercise shows the behaviour of the general form of function

$$M = A - Be^{kt} \qquad (0 \le t \le T).$$

This function decreases at an increasing rate as t increases, but the decrease is from an initial value $(A - B)$.

3. Growth and Decay to a limit

Consider a function intended to represent the speed of a parachutist after the opening of the parachute where v ms^{-1} is the instantanous speed at time t s. An appropriate function may be written

$$v = 12 - 8e^{-1.25t} \qquad (t \geq 0),$$

Let's look at some of the properties and modelling implications of this function. Consider first the value of v when $t = 0$:

$$v = 12 - 8e^{0} = 12 - 8 = 4$$

The function predicts that the parachutist is moving at 4 ms^{-1} when the parachute opens. Consider next the value of v when t is arbitrarily large. For such a value of $t, e^{-1.25t}$ would be arbitrarily small, so v would be very close to the value 12. The modelling interpretation of this is that eventually the speed is very close to a constant value, 12 ms^{-1} which will be maintained until the parachutist lands.

Incidentally, the steady speed which is approached by the parachutist (or anything else falling against air resistance), is called the *terminal velocity*. The parachute, of course, is designed to ensure that the terminal velocity is sufficiently low (12 ms^{-1} in the specific case we have looked at here) to give a reasonably gentle landing and avoid injury.

Now consider what happens as t increases from near zero. When t is near zero, the speed will be near 4 ms^{-1}. The amount being subtracted from 12, through the term 8e$^{-1.25t}$, is close to 8. As t increases the value of 8e$^{-1.25t}$ decreases fairly rapidly at first and then more gradually until v is very nearly 12. The result of this is sketched in Figure 6.5. In fact v is never equal to 12 but gets as close as anyone would like as t increases. The value shown as a horizontal broken line in Figure 6.5 is called an asymptotic limit for v.

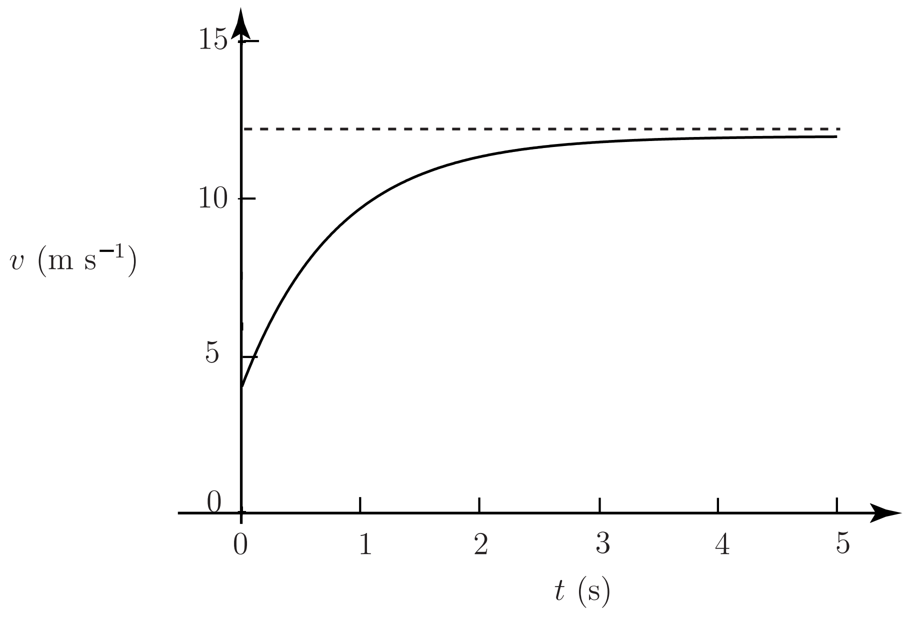


Figure 6.5 Graph of a parachutist's speed against time

We have considered modelling the approach of a parachutist's velocity to terminal velocity but the kind of behaviour portrayed by the resulting function is useful generally in modelling any *growth to a limit*.

A general form of this type of 'growth-to-a-limit' function is

$$y = a - be^{-kx} \qquad (C \leq x \leq D)$$

where a, b and k are positive constants (parameters) and C and D represent values of the independent variable between which the function is valid. Let's check on the properties of this general function. When $x = 0, y = a - be^0 = a - b$. As x increases the exponential factor e^{-kx} gets smaller, so y will increase from the value $a - b$ but at an ever-decreasing rate. As be^{-kx} becomes very small, y, approaches the value a. This value represents the limit, towards which y grows. If a function of this general form was being used to create a model of *population* growth to a limit, then a would represent the limiting population, and $a-b$ would represent the starting population.

There are three parameters, a, b, and k in the general form. Knowledge of the initial and limiting population only gives two pieces of information. A value for the population at some non-zero time is needed also to evaluate the third parameter k.

As an example let's find a function to describe a food-limited bacterial culture that has 300 cells when first counted, has 600 cells after 30 minutes but seems to have approached a limit of 4000 cells after 18 hours. We should start by assuming the general form of limit-to-growth function for the bacteria population, with time measured in hours

$$P = a - be^{-kt} \qquad (0 \leq t \leq 18).$$

When $t = 0$ (at the start of counting), $P = 300$. Since the general form gives $P = a - b$ when $t = 0$, this means that

$$a - b = 300.$$

The limit of P according to the general form is a, so $a = 4000$. From this and the value of $a - b$, we have that $b = 3700$. Finally we can use the information that $P = 600$ when t (measuring time in hours) $= 0.5$. Substitution in the general form gives

$$600 = 4000 - 3700e^{-0.5k}$$

$$3400 = 3700e^{-0.5k}$$

$$\tfrac{3400}{3700} = e^{-0.5k}$$

Taking natural logs of both sides:

$$k = -2\ln(\tfrac{34}{37}) = 0.1691$$

Note, as a check, that k turns out to be positive as required for a limit-to-growth behaviour. Finally the required function may be written

$$P = 4000 - 3700e^{-0.1691t} \qquad (0 \leq t \leq 18).$$

As a check we should try $t = 18$ in this equation. The result is $P = 3824$ which is close to the required value of 4000.

(a) Find a function that could be used to model the growth of a population that has a value of 3000 when counts start, reaches a value of 6000 after 1 year but appears to be approaching a limit of 12000 after a period of 10 years.

(b) Sketch this function.

Your solution

(a) Start with

$$P = a - be^{-kt} \qquad (0 \leq t \leq 10).$$

where P is the number of members in the population at time t years. The given data suggest that a is 12000 and that $a - b = 3000$. so

$$b = a - 3000 = 12000 - 3000 = 9000.$$

The corresponding curve must pass through $(t = 1,\ P = 6000)$ so

$$6000 = 12000 - 9000e^{-k}$$

$$\mathrm{e}^{-k} = \tfrac{12000-6000}{9000} = \tfrac{2}{3}$$

$$-k = \ln(\tfrac{2}{3})$$

$$k = -\ln(\tfrac{2}{3}) = \ln(\tfrac{3}{2})$$

So the population function is

$$P = 12000 - 9000e^{-\ln(\frac{3}{2})t} \qquad (0 \leq t \leq 10).$$

Note that $P(10)$ according to this formula is approximately 11840, which is reasonably close to the required value of 12000.

(b) See Figure 6.6

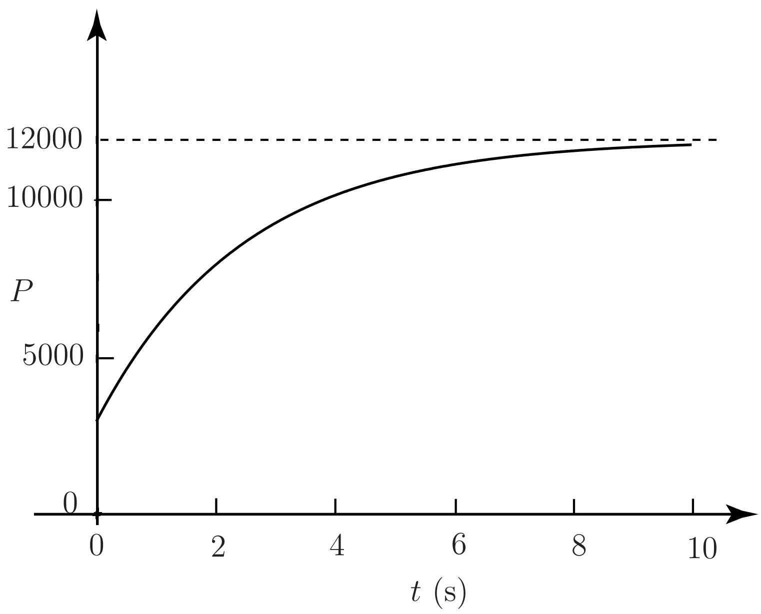
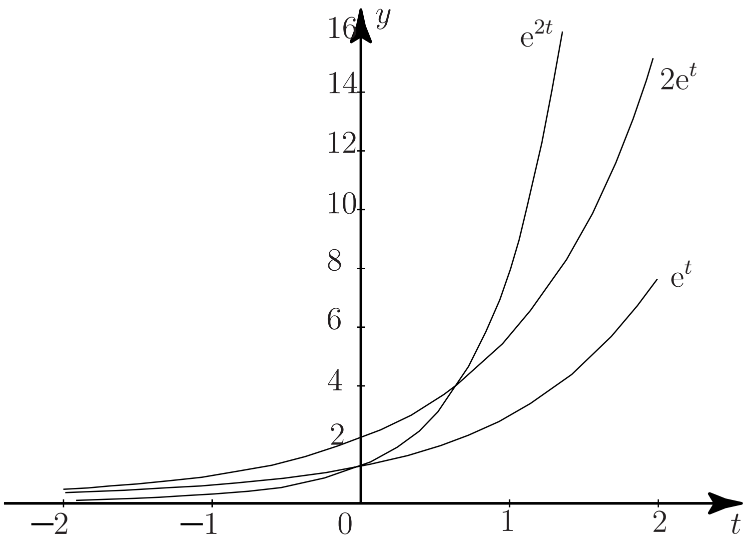


Figure 6.6

Exercises

1. Sketch the graphs of (a) $y = \mathrm{e}^t$ (b) $y = \mathrm{e}^t + 3$ (c) $y = \mathrm{e}^{-t}$ (d) $y = \mathrm{e}^{-t} - 1$

2. The accompanying figure shows on the same axes the graphs of $y = \mathrm{e}^t$, $y = 2\mathrm{e}^t$ and $y = \mathrm{e}^{2t}$.

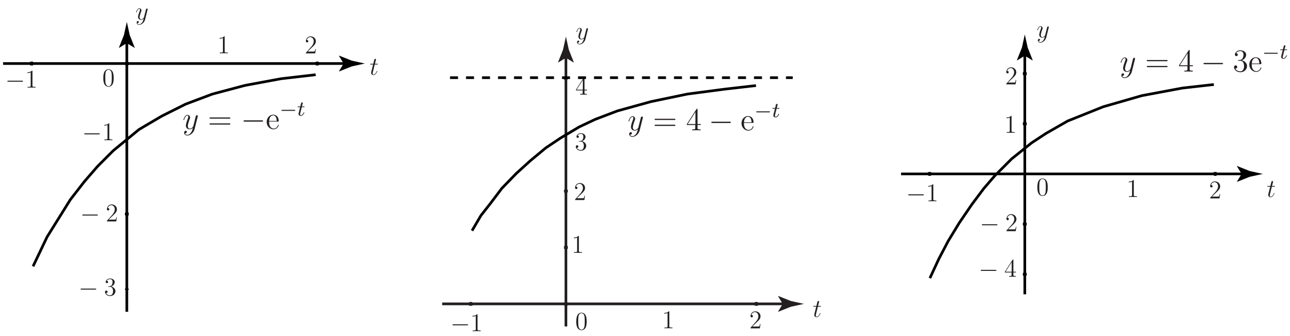


   (a) State in words how the graph of $y = 2\mathrm{e}^t$, $y = \mathrm{e}^{2t}$ relates to the graph of $y = \mathrm{e}^t$.

   (b) Sketch on the same axes graphs of $y = \mathrm{e}^{-t}$, $y = 3\mathrm{e}^{-t}$, $y = \mathrm{e}^{-3t}$.

3. The accompanying figure shows graphs of $y = -\mathrm{e}^{-t}$, $y = 4 - \mathrm{e}^{-t}$ and

   $y = 4 - 3\mathrm{e}^{-t}$. Sketch graphs of: (a) $y = 5 - \mathrm{e}^{-t}$ (b) $y = 5 - 2\mathrm{e}^{-t}$

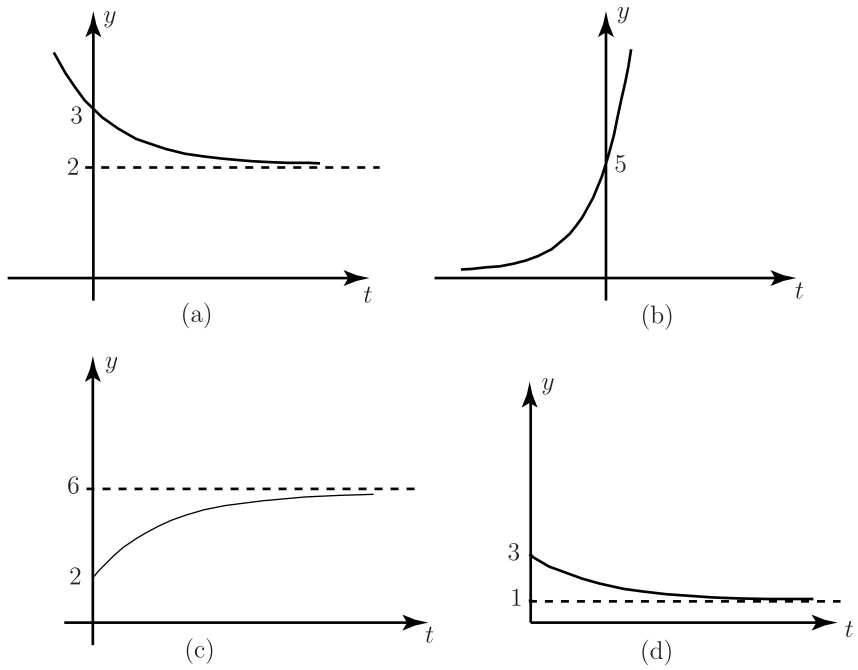


4. (a) The graph (a) in the accomplanying Figure has an equation of the form

   $y = A + \mathrm{e}^{-kt}$, where A and k are constants. What is the value of A?

   (b) The graph (b) has an equation of the form $y = \mathrm{e}^{kt}$ where A and k are constants. What is A?

   (c) Write down the form of the equations of the graphs in (c) and (d) giving numerical values to as many constants as possible.



Note that the function in 4(a) corresponds to decay-to-a-limit.

4. Logarithmic Functions

Experimental psychology is concerned with observing and measuring human response to various stimuli. In particular, our sensations of light, colour, sound, taste, touch and muscular tension are produced when an external stimulus acts on the associated sense. Gustav Fechner, a German scientist of the late nineteenth century, studied the results of experiments involving sensations of heat, light and sound and associated stimuli produced by another German called Ernst Weber. Weber measured the response of subjects, in a laboratory setting, to input stimuli measured in terms of energy or some other physical attribute and discovered that:

(1) No sensation is felt until the stimulus reaches a certain value, known as the threshold value.

(2) After this threshold is reached an increase in stimulus produces an increase in sensation.

(3) This increase in sensation occurs at a diminishing rate as stimulus is increased.

(a) Do Weber's results suggest a linear or non-linear relationship between sensation and stimulus? Sketch a graph of sensation against stimulus according to Weber's results.

(b) Consider whether an exponential function or a limit-to-growth function might be appropriate.

(a) Non-linearity is required by observation (3). See Figure 6.7.

(b) An exponential-type of growth is not appropriate for a model consistent with these experimental results, since we need a diminishing rate of growth in sensation as the stimulus increases. A growth-to-a-limit type of function is not appropriate since the data, at least over the range of Weber's experiments, do not suggest that there is a limit to the sensation with continuing increase in stimulus; only that the increase in sensation occurs more and more slowly.

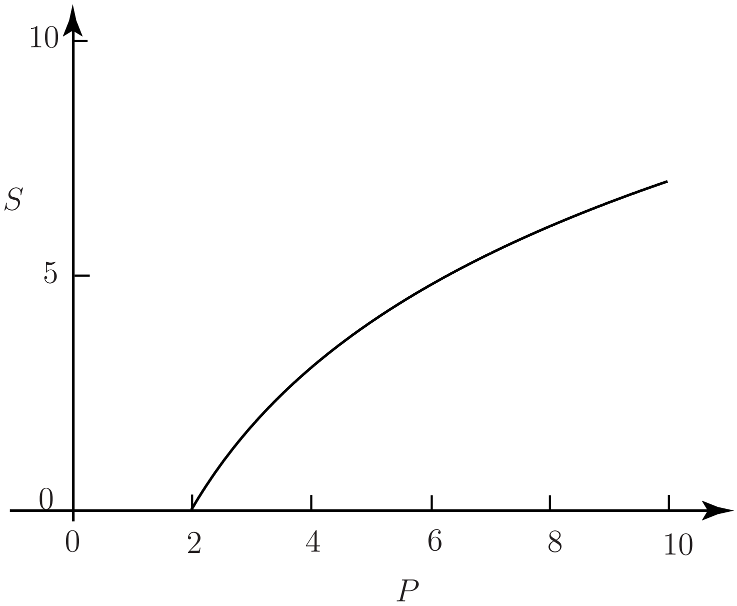


Figure 6.7 Part of Answer to Exercise 6.8 (a)

Fechner suggested that an appropriate function would be logarithmic. He suggested that the variation in sensation (S) with the stimulus input (P) is

$$S = A\log(P/T) \qquad (1 \geq T > 0)$$

where T represents the threshold of stimulus input below which there is no sensation and A is a constant. Note that when $P = T, \log(P/T) = \log(1) = 0$, so this function is consistent with item (1) of Weber's results. Recall also that log means logarithm to base 10, so when $P = 10T, S = A\log(10) = A$. When P = 100T, $S = A\log(100) = 2A$. The logarithmic function predicts that a tenfold increase in the stimulus input from T to $10T$ will result in the same change in sensation as a further tenfold increase in stimulus input to $100T$. Each tenfold change is stimulus results in a doubling of sensation. so, although sensation is predicted to increase with stimulus, the stimulus has to increase at a faster and faster rate to achieve a given change in sensation. These points are consistent with items (2) and (3) of Weber's findings. Fechner's suggestion, that the logarithmic function is an appropriate one for a model of the relationship between sensation and stimulus, seems reasonable. Note that the logarithmic function suggested by Weber is not defined for zero stimulus but we are only interested in the model at and above the threshold stimulus, i.e. for values of the logarithm equal to and above zero, anyway. Note also that the logarithmic function is useful for looking at changes in sensation relative to stimulus values other than the threshold stimulus. According to rule 2 in Table 6.2, Fechner's sensation function may be written

$$S = A\log(P/T) = [\log(P) - \log(T)] \qquad (P \geq T > 0).$$

Suppose that the sensation has the value S_1 at P_1 and S_2 at P_2, so that

$$S_1 = A[\log(P_1) - \log(T)] \qquad (P_1 \geq T > 0),$$

and

$$S_2 = A[\log(P_2) - \log(T)] \qquad (P_2 \geq T > 0).$$

If we subtract the first of these two equations from the second, we get

$$S_2 - S_1 = A[\log(P_2) - \log(P_1)] = A\log(P_2/P_1),$$

where rule 2 of Table 6.2 has been used again for the last step. According to this form of equation, the change in sensation between two stimuli values depends on the ratio of the stimuli values.

Another point to note is that the relationship between the variables in a logarithmic function is really the *reverse* of that between the variables in the exponential function.

Let's start with

$$S = A\log(P/T) \qquad (1 \geq T > 0).$$

Divide both sides by A.

$$\tfrac{S}{A} = \log\tfrac{P}{T} \qquad (1 \geq T > 0).$$

Undo the logarithm on both sides by raising 10 to the power of each side:

$$10^{S/A} = 10^{\log(P/T)} = \frac{P}{T} \qquad (1 \geq T > 0).$$

Use rule 8 from Table 6.2

$$10^{S/A} = \frac{P}{T} \qquad (1 \geq T > 0).$$

or $P = T10^{S/A} \qquad (1 \geq T > 0).$

This is an exponential relationship between stimulus and sensation. A logarithmic relationship between sensation and stimulus therefore implies an exponential relationship between stimulus and sensation. The relationship may be written in two diferent forms with the variables playing opposite rules in the two functions.

The logarithmic relationship between sensation and stimulus is known as the *Weber-Fechner Law of Sensation.* The idea that a mathematical function could describe our sensations was quite startling when it was first propounded. Indeed it may seem quite amazing to you now. Moreover it doesn't always work. Nevertheless the idea has been quite fuitful. Out of it has come much quantitative experimental psychology. For example it relates to the sensation of the loudness of sound. Sound level is expressed on a logarithmic scale. At a frequency of 1 kHz an increase of 10 dB corresponds to a doubling of loudness.

Given a relationship between y and x of the form $y = 3\log(\frac{x}{4}) \qquad (x \geq 4)$, what is the relationship between x and y?

One way of answering is to compare with the example preceding this activity. We have y in place of S, x in place of P, 3 in place of A, 4 in place of T. So it is possible to write down

$$x = 4 \times 10^{y/3} \qquad (y \geq 0)$$

Alternatively we can manipulate the given expression algebraically, starting with $y = 3\log(\frac{x}{4})$, divide both sides by 3 to give $y/3 = \log(x/4)$. Raise 10 to the power of each side so that $10^{y/3} = x/4$. Multiply both sides by 4 and rearrange, to obtain $x = 4 \times 10^{y/3}$, as before. The associated range is the result of the fact that $x \geq 4$, so $10^{y/3} \geq 1$, so $y/3 > 0$ or $y > 0$.