# Vectors & Electrostatics

9.6

## Introduction

needs doing

## Prerequisites

Before starting this Section you should ...

①

## Learning Outcomes

After completing this Section you should be able to ...

✓

Electric charge is measured in Coulombs (C). Charges can be either positive or negative. The Coulomb force between two charges in free space is:

$$\vec{F} = \frac{Q_1 \times Q_2}{4\pi\epsilon_0|\vec{R}|^2}\frac{\vec{R}}{|\vec{R}|} \tag{1}$$

where $Q_1$ and $Q_2$ are the two electric charges and $P_1$ and $P_2$ are their positions, so that $\vec{R} = |P_1 - P_2|$ is the separation between the two charges. $\epsilon_0 = 8.85 \times 10^{12}$ Farads/metre (F/m) and the force $F$ is measured in Newtons (N). $F$ has both magnitude and direction (along the line vector joining the positions of the two charges in co-ordinate space).

## The electric field

The force experienced by an electric charge of unit magnitude at $P_2$, due to the presence of another charge $Q_1$ Coulombs (C) located at $P_1$, is the electric field $\vec{E}$ Newtons per Coulomb (N/C or V/m) at $P_2$, is

$$\vec{E} = \frac{Q_1}{4\pi\epsilon_0|\vec{R}|^2}\frac{\vec{R}}{|\vec{R}|} \tag{2}$$

Therefore, the total electric field $G$ due to point charges $Q_1$ at $P_1$, $Q_2$ at $P_2$ etc is:

$$\vec{E} = \frac{Q_1}{4\pi\epsilon_0|\vec{R}_{GP_1}|^2}\frac{\vec{R}_{GP_1}}{|\vec{R}_{GP_1}|} + \frac{Q_1}{4\pi\epsilon_0|\vec{R}_{GP_2}|^2}\frac{\vec{R}_{GP_2}}{|\vec{R}_{GP_2}|} + \cdots \tag{3}$$

where $\vec{R}_{GP_1}$ is the vector joining positions $G$ and $P_1$, etc.

## The work done

The work done $W$ (energy expended) in moving a charge $q$ through a distance d$S$ , in a direction given by the unit vector $\vec{S}/|\vec{S}|$ , in an electric field $\vec{E}$ is

$$W = -q\vec{E}.\mathrm{d}\vec{S} \tag{4}$$

where $W$ is in Joules.

## Exercises

1. In free space point charge $Q_1 = 10\ \mu$C is at $P_1(0, -4, 0)$ and charge $Q_2 = 20\ \mu$C is at $P_2 = (0, 0, 4)$ (see Figure 1).

   (a) Find $E$ at the origin.

   (b) Where should a third charge $Q_3 = 30\ \mu$C be placed so that the field $E$ is zero at the origin?

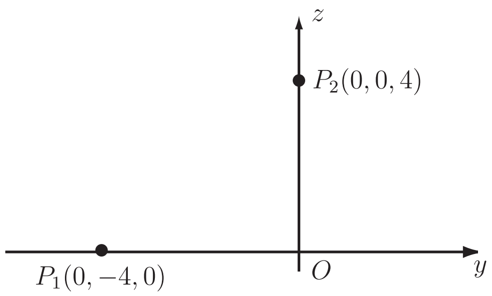


Figure 1

2. Eight point charges of 1 $\mu$C each are located at the corners of a cube in free space which is 1m on each side (see Figure 2). Calculate $|E|$ at the centre of

   (a) the cube,

   (b) a face, and

   (c) an edge.

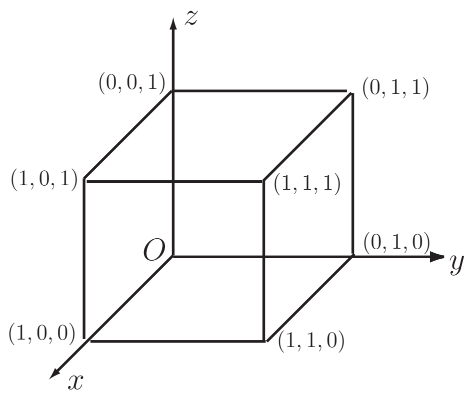


Figure 2

3. If $\vec{E} = -50y\hat{x} - 50x\hat{y} + 30\hat{z}$ V/m, where $\hat{x}, \hat{y}$ and $\hat{z}$ are unit vectors in the $x, y$ and $z$ directions respectively, find the differential amount of work done in moving a 2 $\mu$C point charge a distance of 5mm from (a) $P(1, 2, 3)$ towards $Q(2, 4, 1)$ and (b) from $Q(2, 4, 1)$ towards $P(1, 2, 3)$.

1. (a) Total field at the origin $E =$ (field at origin due to charge at $P_1$) + (field at origin due to charge at $P_2$). Therefore

$$\vec{E}_{1,2} = \frac{10 \times 10^{-9}}{4\pi 8.854 \times 10^{-12} 4^2}\hat{y} + \frac{20 \times 10^{-9}}{4\pi 8.854 \times 10^{-12} 4^2}(-\hat{z})$$
$$= 5.62\hat{y} - 11.21\hat{z}$$

(The negative sign in front of the second term indicates that the field due to the charge at $P_2(0, 0, -4)$ at origin is in reverse direction to the co-ordinate axis.)

(b) Suppose the third charge $Q_3 = 30\ \mu$C is placed at $P_3(0, -a, b)$. The field at the origin due to the third charge is

$$\vec{E}_3 = \frac{30 \times 10^{-9}}{4\pi 8.854 \times 10^{-12}(a^2 + b^2)} \frac{(-a\hat{y} + b\hat{z})}{(a^2 + b^2)^{1/2}}$$

If the position of the third charge is such that the total field at the origin is zero, then $E_3 = -E_{1,2}$ . There are two unknowns ($a$ and $b$) with two equations.
We find $P_3(0, 2.07, -4.14)$.

2. (a) The field at the centre of the cube is zero because of the symmetrical distribution of charges.

(b) When calculating the total field at the centre of one of the faces of the cube, the electric field from the four corners in the plane of the face considered cancel each other. The other four charges are $|\vec{R}| = \sqrt{1.5}$m from the centre of a face. Therefore, the total field at the centre of the face of the cube is:

$$E_{\text{face}} = \frac{10^{-9}}{4\pi 8.854 \times 10^{-12}(1.5)3/2}(\vec{R}_1 + \vec{R}_2 + \vec{R}_3 + \vec{R}_4) \qquad \text{Ans. } = 19.92\ \text{Vm}^{-1}$$

(c) 25.72 $\text{Vm}^{-1}$

3. (a) The work done in moving a 2 $\mu$C charge through a distance of 5mm from $P$ towards $Q$ is

$$W = -(2 \times 10^{-6}) \times (5 \times 10^{-6})\vec{E}\frac{\vec{R}_{PQ}}{|\vec{R}_{PQ}|}$$

$$= 10^{-11}(-50y\hat{x} - 50x\hat{y} + 30\hat{z})\frac{(-\hat{x} - 2\hat{y} + 2\hat{z})}{\sqrt{1 + 2^2 + 2^2}}$$

Substituting the values of $x, y$ and $z$ from the co-ordinates at $P$, we find $W(P) = -2600/3$ J.

(b) A similar calculation yields that the work done in moving the same charge through the same distance from $Q$ to $P$ is $W(Q) = 4600/3$ J.